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Differential awareness, ambiguity, and incomplete contracts: A model of contractual disputes☆

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ABSTRACT

We focus on aspects of differential awareness that give rise to contractual disputes. Parties to a contract are boundedly rational as the state space available to them is coarser than the complete state space. Hence, they may disagree as to which state of the world has occurred, and therefore as to what actions are required by the contract. Such disagreement leads to disputes. We show that the agents may prefer simpler less ambiguous contracts when facing potential disputes.

1. Introduction

A complete contract between unboundedly rational parties should specify, for each possible state of the world, an action for each party. No dispute should arise, since any potential source of dispute should have been anticipated in the design of the contract. In reality, however, contracts do not completely specify the actions required of the parties and disputes take place regularly.

The idea that incompleteness in contracts arises from an inability to specify and contract on the state space is not new; it has been a standard argument at least since Williamson (1975, 1985) drew attention to the importance of transactions costs in determining contractual structures. These transactions costs are typically imputed to incompleteness of the state space. However, as Maskin and Tirole (1999) observe, incompleteness of the state space is not, in itself, sufficient to preclude the achievement of the first best contract. They conclude (p. 106) that '[i]f we are to explain "simple institutions"such as property rights, authority (or more generally, decision processes), short-term contracts and so forth, a theory of bounded rationality is certainly an important, perhaps ultimately essential ingredient'.

Recent developments in the analysis of unawareness and differential awareness (Board and Chung, 2007, 2009; Heifetz et al., 2006, 2010; Halpern and Rêgo, 2006, 2008; Li, 2009; Grant and Quiggin, 2012) have laid the basis for a theory of

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contractual incompleteness based on bounded rationality. In this literature, the unrealistic assumption that individuals can consider, and plan for, all welfare-relevant contingencies is replaced by models in which awareness is bounded and evolves dynamically as unforeseen contingencies are realized. In particular, in models of interactive differential unawareness, standard, but computationally demanding and implausible, assumptions about common knowledge are dropped. Individuals are assumed both to be boundedly aware and to impute bounded rationality to others.

In this paper, we focus on aspects of differential awareness that give rise to contractual disputes. By differential awareness, we mean that the parties to the contract may be aware of different things. Our modeling technique will be to treat the unawareness in a reduced form described by dispute relations, which transforms the problem into one of ambiguity. While this differs from approaches to unawareness in the line of Heifetz et al. (2006), it is a tractable alternative for attacking contractual disputes. We model boundedly rational agents as reasoning with respect to a subjective state space that is coarser than the completely specified state space in which all welfare-relevant contingencies are represented. Furthermore, even though different agents may use a common language to describe a contract, their semantic interpretations of that language may differ. Hence, they may disagree as to which state of the world has occurred, and therefore as to what actions are required by the contract. Such disagreement will lead to contractual disputes. If agents understand differential awareness, they may prefer simple and unambiguous contracts to more fully specified contracts that are more liable to generate disputes.

Formally, we represent agents' understanding of differential awareness using the Gul and Pesendorfer (2010) expected uncertain utility model and the assumption that disputes give rise to a 'war of attrition' game in which both parties receive the (mixed-strategy equilibrium) expected utility equal to that associated with their least-favored interpretation of the world. Our approach establishes a connection between aversion to semantic ambiguity (the sense in which the term 'ambiguity' is normally found in ordinary usage) and state-contingent ambiguity (the sense in which the term is commonly used in decision theory).

Given these preferences, we show that for a two-agent bargaining process over risk-sharing contracts, an individually rational and efficient contract involves a trade-off between risk and ambiguity. A finer contractual specification increases the gains from risk sharing when the contract is implemented successfully, but also increases the ambiguity of the contract and creates more possibilities for dispute. In this context, we find that risk aversion makes agents more likely to engage in contracts involving ambiguous terms and discuss the trade off between risk aversion and willingness to contract in the face of ambiguity.

The paper is organized as follows. We begin in Section 2 with an illustrative example that will be used to present results throughout the paper. In Section 3, we lay out the framework in which contracts are specified and then develop the concept of contractual ambiguity. In Section 4 we adopt Gul and Pesendorfer's expected uncertain utility model and derive preferences over ambiguous contracts. In Section 5 we formulate the associated bargaining problem and characterize the set of individually rational and efficient contracts. In Section 6 we discuss the implications of our analysis and its relationship to the existing literature on incomplete contracts and bounded rationality.

2. An illustrative example

In informal discussions of ambiguous contracts, it is common to refer to 'gray areas'. Some contracts, or contingencies specified in contracts, are seen as having gray areas, thereby giving rise to possibilities of disagreement and dispute, while others are seen as relatively clear-cut and unambiguous.

We develop these ideas in an example.[1] Suppose two individuals, player 1 and player 2 contemplate entering into a risk-sharing contract. They will draw a card from a pack. The card may be all white, all black, all red or it may be white at the top and black at the bottom. From the viewpoint of a fully informed outside observer there are four possible states of the world, one for each card.

Each player sees the world as white, black or red. However, player 1 although not explicitly aware of this, always observes the bottom half of the card. Player 2 always observes the top half, although again he is not explicitly aware of the 'orientation' of his view. Thus, if the card is white at the top and black at the bottom, player 1 will construe the card is black, while player 2 will construe it as white. The underlying state space and the two individuals' partitions of the black–white–red spectrum are summarized in Table 1, where X denotes a pair of observations that is inconsistent with the problem description and therefore does not correspond to a state. Suppose the state-contingent endowments of the two individuals are given in the bi-matrix of Table 2.

Each individual faces a single source of uncertainty that is measurable with respect to his or her own partition of the state space. We assume that both players are risk-averse and view the three elements of their respective partitions as 'exchangeable' (Chew and Sagi, 2006).[2] Hence both parties would prefer the non state-contingent allocation (2, 2) in every

[1] We are indebted to Bob Brito for suggesting this example.

[2] In this context, 'exchangeable' is equivalent to each individual being indifferent between betting on any element of his or her partition.

Table 1
Observations.

		Player 2's observation		
	Card drawn is:	*white (at top)*	*black (at top)*	*red (at top)*
Player 1's observation	*white (at bottom)*	white white	X	X
	black (at bottom)	white black	black black	X
	red (at bottom)	X	X	red red

state. So, ignoring (for the moment) any possibility of future disagreement and dispute, both would find it attractive to sign a risk-sharing contract of contingent transfers from 2 to 1:

$$\tilde{c} = \begin{cases} 1 & \text{if the card drawn is black} \\ -1 & \text{if the card drawn is red} \\ 0 & \text{otherwise.} \end{cases} .$$

In the formal framework developed below, if such a contract were signed, the presumption is that each party will assess which contingency has obtained according to her or his own semantics. For player 1, this entails assessing that 'the card drawn is black' is true when she makes the observation the card drawn 'is black' which occurs whenever the card drawn is black at the bottom. Player 2, on the other hand assesses that 'the card drawn is black' is true when he observes the card drawn 'is black' which occurs whenever the card drawn is black at the top.

The card that is white at the top and black at the bottom creates a possibility for disagreement since player 1 will interpret this as 'black', and so believe that she is entitled to receive a payment of 1. Player 2 will in the same situation interpret this as 'white', so he will expect no payment is required. Hence, a disagreement will ensue.

The awareness of the players includes a number of elements. First, each player is aware of their own state-contingent description of the world and of the information available to them. Second, given the description above, each is aware that the other may not have access to their model of the world. In this example, player 1 and player 2 are both aware that each of them is aware of the statements 'the card is white', 'the card is black' and 'the card is red'. However, each is also conscious that their model and the model of the other individual may be incomplete. In particular there may exist other details about the world of which neither is currently aware, that lead to different interpretations by the two about the semantic content of those statements for the two players. Thus, the central feature of the example is that players are boundedly rational, but nonetheless sophisticated enough to reason about their own bounded rationality and that of others.

In this setup, boundedly rational players are unable to formulate a description sufficiently refined to allow the contract to specify a resolution in the case of a dispute. Specifically, in the illustrative example, the two players are unaware that they are viewing the card from a different orientation. However, they understand that disputes are possible. Depending on the weight they place on this possibility, they may choose a contract which offers only partial hedging, or they may choose not

Table 2
Endowments.

		Player 2's endowment		
	Card drawni s:	*white (at top)*	*black (at top)*	*red (at top)*
Player 1's endowment	*white (at bottom)*	2 2	X	X
	black (at bottom)	2 1	3 1	X
	red (at bottom)	X	X	1 3

to contract at all. This corresponds closely to the risk–uncertainty distinction of Knight (1921) whose main concern was with uncertainties that could not be hedged through market contracts such as insurance, and therefore reduced to manageable risk. Uncertainty of this kind was central to Knight's idea of entrepreneurship.

Other forms of bounded rationality have been considered in the literature. A number of analyses have suggested that contractual incompleteness may arise when agents do not satisfy the stringent rationality assumptions of expected utility theory. Segal (1999) has argued that, in some complex environments, distinctions between complete and incomplete contracts might become trivial. Mukerji (1998) and Mukerji and Tallon (2001) discussed incomplete contracts in the presence of the decision-theoretic concept of 'ambiguity', which refers to a situation in which an agent's preferences cannot be rationalized by a specific probability distribution over a commonly known state space. Board and Chung (2007, 2009) develop an object-based model of unawareness, and consider the problems that arise when one party understands that they are less aware than the party they are contracting with. Filiz-Ozbay (2010) similarly deals with cases where one party (the insurer) is more aware than the other (the insured).

3. Language, contracts and ambiguity

The central concern of this paper is to develop a model of contracting between parties whose bounded rationality is embodied in the ambiguity of the interpretations of the language they use to describe the world. We do not formally model the awareness and knowledge of the players. As mentioned in the introduction, we will use a reduced form representation which translates the problem into one of ambiguity. The reduction takes the form of what we call dispute relations for each party to the contract. While these relations generate dispute sets that look like information sets, they are not used to determine awareness or knowledge. We require only that individuals have access to different subjective state spaces and that they understand this fact about themselves and others, even if this understanding cannot itself be represented within the subjective state space available to them.

We consider two parties $i = 1, 2$ who implicitly have access to a common language in which contracts can be written. Generically, a contract lists a specific action required to be performed by the parties contingent on some set of circumstances having obtained. More explicitly, we denote by S the finite set of all mutually excludable and exhaustive contingencies that can be expressed in the common language, and by 2^S the set of all collections of contingencies (that is, events).[3] We denote by A, the set of actions that are expressible in the common language, where a typical action $a \in A$ might be 'player i performs service z for player j in return for consideration w'. We take A to be a compact and convex subset of a separable metric space. We denote by C, the set of expressible contracts, which we identify formally with the set of mappings $c : S \to A$ with finite range.

To relate this to the example discussed in Section 2, we can set $S := \{\boldsymbol{W}, \boldsymbol{B}, \boldsymbol{R}\}$. The contingency $\boldsymbol{W}$ (respectively, $\boldsymbol{B}$, $\boldsymbol{R}$) corresponds to the contingency in which the card drawn is 'white' (respectively, 'black', 'red'). The set of actions is the set of transfers from individual 2 to individual 1, $A = [-3, 3]$. The set of contracts can thus be characterized by contingent transfers $(A)^S$ and the contingent endowments are given by:

	Contingency		
Ind.	$\boldsymbol{W}$	$\boldsymbol{B}$	$\boldsymbol{R}$
z_s^1	2	1	3
z_s^2	2	3	1

Because we have chosen formally identical contingency spaces for the players, implicitly the test-interpretation of each player and the language of each player are identical. The distinction and the source of disputes is thus purely semantic. Disputes arise from the players disagreeing about the contingency that has obtained, or, equivalently, which underlying tests have been satisfied. We model each players' (common) perception of the *possibility* of disputes by a pair of reflexive relations over the space S, that jointly satisfy a coherency property we call *complementary symmetry*. Formally,

Definition 1. For each $i = 1, 2$, let $\mathcal{D}^i \subset S \times S$ denote the dispute possibility relation for party i. Each $\mathcal{D}^i$ satisfies *reflexivity*: that is, for each s in S, $(s, s) \in \mathcal{D}^i$; and, jointly they satisfy *complementary symmetry*: that is, for any pair of contingencies s and s' in S, $(s, s') \in \mathcal{D}^i \Rightarrow (s', s) \in \mathcal{D}^{3-i}$. We refer to the set $\mathcal{D}^i(s) = \{s' \in S : (s, s') \in \mathcal{D}^i\}$ as the *possibility-of-dispute set for i* associated with contingency s.

The interpretation of the ordered pair (s, s') being in $\mathcal{D}^i$, is that if player i assesses s as having obtained then she thinks it is possible that player $(3 - i)$ has assessed that s' has obtained. Reflexivity is natural since the players are using the same language. It requires that no matter what contingency a player thinks has obtained, she should consider it possible that the other will agree that contingency has indeed obtained.

Complementary symmetry needs more explanation. It can be seen as a joint coherency property. Suppose that after i assesses that s has obtained, he thinks that $(3 - i)$ might assess that s' occurred. Then complementary symmetry requires

[3] For example if both parties had access to a non-empty finite set of primitive test propositions $T = \{t_1, \ldots, t_K\}$ then we could set $S := 2^T$, with the interpretation that for each $\hat{T} \in S$ (and hence $\hat{T} \subset T$), $\hat{T}$ corresponds to the contingency in which the primitive test t is true if and only if $t \in \hat{T}$.

that, in turn, after $(3-i)$ assesses s' obtained, he thinks it possible that i assesses s has obtained. In a symmetric world like the one we have chosen, this condition is natural. However, if the two parties are coming from quite different backgrounds, e.g., one is an insurance company and the other is a consumer of insurance, then the assumption might be less natural. In fact, the assumption that the parties have the same state space S might be invalid.[4]

For each $s \in S$, the set $\mathcal{D}^i(s)$ is the collection of possible contingencies the other player may have determined as having obtained when i determines that contingency s has obtained.

We can use the possibility of disputes to derive the sets of contingencies that are unambiguous.

Definition 2. The set of *unambiguous events* $\mathcal{E}_U \subseteq 2^S$ is given by:

$$\mathcal{E}_U = \left\{ E \subseteq S : \bigcup_{s\in E} \mathcal{D}^1(s) = \bigcup_{s\in E} \mathcal{D}^2(s) = E \right\}.$$

The set of *ambiguous events* $\mathcal{E}_A = 2^S - \mathcal{E}_U$.

Lemma 1. *The set of unambiguous events $\mathcal{E}_U$ is an algebra of subsets of S, that is, it is non-empty, and closed under taking complements and intersection.*

Proof. First notice that, by reflexivity, it follows that for any $E \subseteq S$, and both $i = 1, 2$, $E \subseteq \bigcup_{s\in E} \mathcal{D}^i(s)$. So, in particular, $S \subseteq \bigcup_{s\in S} \mathcal{D}^i(s)$. But since $\bigcup_{s\in E} \mathcal{D}^i(s) \subseteq S$, it follows that $\bigcup_{s\in S} \mathcal{D}^1(s) = \bigcup_{s\in S} \mathcal{D}^2(s) = S$, that is, $S \in \mathcal{E}_U$. $\varnothing \in \mathcal{E}_U$ follows by the convention $\bigcup_{s\in\varnothing} \mathcal{D}^1(s) = \varnothing$.

To see that $\mathcal{E}_U$ is closed under taking complements, suppose $E \subset S$ is in $\mathcal{E}_U$. By reflexivity it immediately follows $S \backslash E \subseteq \bigcup_{s'\notin E} \mathcal{D}^i(s')$, for both $i = 1, 2$. It remains to show that $\bigcup_{s'\notin E} \mathcal{D}^i(s') \subseteq S\backslash E$, for both $i = 1, 2$. To do this fix $s' \notin E$ and $s'' \in \mathcal{D}^i(s')$ (that is, $(s', s'') \in \mathcal{D}^i$.) Suppose, contrary to what we are trying to show, that $s'' \in E$. By complementary symmetry $(s'', s') \in \mathcal{D}^{3-i}$, that is, $s' \in \mathcal{D}^{3-i}(s'')$. Hence $s' \in \bigcup_{s\in E} \mathcal{D}^{3-i}(s) = E$, since E is unambiguous. But then we have $s' \in E$, a contradiction. Thus we have, $\bigcup_{s'\notin E} \mathcal{D}^1(s') = \bigcup_{s'\notin E} \mathcal{D}^2(s') = S\backslash E$, that is, $S\backslash E \in \mathcal{E}_U$, as required.

It remains to show that $\mathcal{E}_U$ is closed under intersection. Suppose A and B are both in $\mathcal{E}_U$. Since $\mathcal{E}_U$ is closed under taking complements, it follows that both $S \backslash A$ and $S \backslash B$ are in $\mathcal{E}_U$. That is,

$$\bigcup_{s\in S\backslash A} \mathcal{D}^1(s) = \bigcup_{s\in S\backslash A} \mathcal{D}^2(s) = S\backslash A \text{ and} \bigcup_{s\in S\backslash B} \mathcal{D}^1(s) = \bigcup_{s\in S\backslash B} \mathcal{D}^2(s) = S\backslash B.$$

But taking unions we have

$$\left[\bigcup_{s\in S\backslash A} \mathcal{D}^1(s)\right] \cup \left[\bigcup_{s\in S\backslash B} \mathcal{D}^1(s)\right] = \left[\bigcup_{s\in S\backslash A} \mathcal{D}^2(s)\right] \cup \left[\bigcup_{s\in S\backslash B} \mathcal{D}^2(s)\right] = (S\backslash A) \cup (S\backslash B) = S\backslash (A \cap B).$$

or equivalently,

$$\bigcup_{s\in S\backslash(A\cap B)} \mathcal{D}^1(s) = \bigcup_{s\in S\backslash(A\cap B)} \mathcal{D}^2(s) = S\backslash (A \cap B).$$

That is, $S\backslash (A \cap B)$ is in $\mathcal{E}_U$ and since complements are also unambiguous, we have $(A \cap B)$ is in $\mathcal{E}_U$ as required. □

For each $s \in S$ we can define the smallest unambiguous event $E(s)$ containing s by $E(s) := \bigcap_{E\in\{F\in\mathcal{E}_U : s\in F\}} E$. It follows immediately from the definition of unambiguous events that the coarsest common-refinement of $\left\{\mathcal{D}^1(s)\right\}_{s\in S} \cup \left\{\mathcal{D}^2(s)\right\}_{s\in S}$ is the finest unambiguous partition of S.

Notice that if a contract is measurable with respect to the *unambiguous partition*, $\left\{E(s)\right\}_{s\in S}$ although the individuals might disagree about the actual contingency that has obtained, they will never disagree about which action the contract prescribes. Hence such contracts are viewed as *unambiguous.*

Definition 3. A contract is *unambiguous* if for all $s, s' \in S$, $E(s) = E(s') \Rightarrow c(s) = c(s')$. We denote by C_U the set of unambiguous contracts.

4. Preferences over contracts

To model the individual's preferences over contracts, we adopt a special case of the *expected uncertain utility* (EUU) model of Gul and Pesendorfer (2010). While this model is not essential to our approach, we find it appealing for the setting since it relies on the notion that individuals have coarse measurements of some more complete underlying state space. To employ their model, we take the 'state-space' of individual i to be the product space of the individual contingency spaces $S^i \times S^{(3-i)} \equiv \Omega^i$.[5] Preferences $\succsim_i$ are assumed to be defined over acts, that is functions $\mathbf{f}$ from $S^i \times S^{(3-i)}$ to the set of outcomes, which following Gul and Pesendorfer, we take to be an interval of finite length $[m, M]$ of monetary outcomes.

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[4] We thank an anonymous referee for suggesting the insurance example.

[5] Though we keep the assumption that $S = S^1 = S^2$, we use S^i and $S^{(3-i)}$ for clarity.

Since our objects of choice are contracts, we need to have a specification for translating a contract c into an act for each individual i. Let $y^i : A \times S^i \to [m, M]$ be the outcome function where $y^i(a, s)$ is the outcome in $[m, M]$ that results for individual i when action a from A is undertaken and she observes s in S^i. This may be viewed as a private value assumption. It implies that each person does not obtain any welfare relevant information from a dispute. We maintain this assumption for simplicity.

We presume that individual i associates the contract c with the 'act' $\mathbf{f}^i_c : S^i \times S^{(3-i)} \to [m, M]$, where,

$$\mathbf{f}^i_c\left(s, s'\right) = \begin{cases} y^i\left(c\left(s\right), s\right) & \text{if } (*) \text{ holds} \\ y^i\left(c\left(s'\right), s\right) & \text{otherwise} \end{cases} +,$$

where $(*)$ is said to hold if either

$$\begin{aligned} &\text{(i)}\, v^{(3-i)}(y^{(3-i)}(c(s), s') &&> v^{(3-i)}(y^{(3-i)}(c(s'), s'), \\ &\text{or(ii)}\, v^{(3-i)}(y^{(3-i)}(c(s), s') &&= v^{(3-i)}(y^{(3-i)}(c(s'), s') \text{ and } v^i(y^i(c(s), s) > v^i(y^i(c(s'), s). \end{aligned}$$

That is, in evaluating a contract, it is as if individual i presumes that the contract c will be implemented according to individual $(3-i)$'s interpretation, unless individual $(3-i)$ strictly prefers i's interpretation, or is indifferent between the two interpretations and i strictly prefers her own interpretation.

This may be viewed as the reduced form derived from a game in which any dispute results in a mixed strategy equilibrium play of a war of attrition where in each round of the war of attrition, as they are playing a mixed strategy each individual must be indifferent between remaining in the war of attrition or conceding (which in this case corresponds to the implementation of the interpretation that favors her opponent).[6]

We assume that each individual evaluates the act associated with a contract as a special type of EUU maximizer characterized by a 4-tuple $\langle \mathcal{E}^i, \mu^i, v^i, \alpha^i \rangle$. Here, $\mathcal{E}^i$ is an algebra generated from $\mathcal{D}^i$, μ^i is a subjective probability measure on $\mathcal{E}^i$, $v^i : [m, M] \to \mathbb{R}$ is a preference scaling function, and $\alpha^i \in [0, 1]$ is an ambiguity attitude parameter.

We use the possibility relation $\mathcal{D}^i$ to obtain the algebra $\mathcal{E}^i$ generated from the partition

$$\left\{\{s\} \times \mathcal{D}^i(s) : s \in S^i\right\} \cup \{\{s\} \times S^{(3-i)} \backslash \mathcal{D}^i(s) : s \in S^i\}.$$

Since each party presumes the other will see something within her or his own dispute set, we require that the measure μ^i defined on $\mathcal{E}^i$ satisfy the following condition: for all s in S^i, $\mu^i\left(\{s\} \times S^{(3-i)} \backslash \mathcal{D}^i(s)\right) = 0$.

When person i sees s, he is not sure which $s' \in D^i(s)$ will be seen by person $(3-i)$. We assume, following Gul and Pesendorfer (2010), that the decision maker evaluates the act $\mathbf{f}^i_c(s, s')$ on the (measurable with respect to $\mathcal{E}^i$) event $\{s\} \times \mathcal{D}^i(s)$ in terms of the interval of outcomes $[\min_{s' \in \mathcal{D}^i(s)} v^i\left(\mathbf{f}^i_c(s, s')\right), \max_{s' \in \mathcal{D}^i(s)} v^i\left(\mathbf{f}^i_c(s, s')\right)]$. In particular, he assigns the 'utility' $\alpha^i \min_{s' \in \mathcal{D}^i(s)} v^i\left(\mathbf{f}^i_c(s, s')\right) + \left(1 - \alpha^i\right) \max_{s' \in \mathcal{D}^i(s)} v^i\left(\mathbf{f}^i_c(s, s')\right)$ to this interval of outcomes and weights it by the probability $\mu^i(\{s\} \times \mathcal{D}^i(s))$.[7]

Putting this all together we have that an EUU decision maker $\langle \mathcal{E}^i, \mu^i, v^i, \alpha^i \rangle$, with outcome function y^i will evaluate a contract c according to the function:

$$V^i(c) = \sum_{s \in S^i} \mu^i\left(\{s\} \times \mathcal{D}^i(s)\right) \left[\alpha^i \min_{s' \in \mathcal{D}^i(s)} v^i\left(\mathbf{f}^i_c\left(s, s'\right)\right) + \left(1 - \alpha^i\right) \max_{s' \in \mathcal{D}^i(s)} v^i\left(\mathbf{f}^i_c\left(s, s'\right)\right)\right]. \tag{1}$$

In this formulation, the bounded rationality of the players is expressed both by the limitations of contracts to S rather than $S^i \times S^{(3-i)}$, and also in evaluation of the contracts as an EUU decision maker, rather than a standard expected utility maximizer.

4.1. Standard state space approach with common prior and expected utility

An alternative approach (similar to the one taken by Board and Chung, 2009) would be to specify a common prior μ over a commonly known state space $S^1 \times S^2$ and treat the players as expected utility maximizers over the expanded state space. Note that here $\mu(s, s')$ denotes the probability that 1 assesses s obtained and 2 assesses s' obtained. Maintaining our assumption from above that both individuals anticipate that in any state (s_1, s_2) with $s_1 \neq s_2$, a dispute arising from

[6] We thank Roger Myerson for the suggestion that disputes might be viewed as wars of attrition.

[7] That is, we assume this EUU maximizer comes from the subclass that employs the Hurwicz criterion to evaluate each interval of outcomes by a constant convex combination of the "utilities" of the lower and upper bounds of the interval.

their different interpretations results in a mixed strategy equilibrium play of a war of attrition, then the resulting subjective expected utility for individual i may be expressed as:

$$U^i(c)=\sum_{s_1\in S^1}\sum_{s_2\in S^2}\mu(s_1,s_2)v^i\left(\mathbf{f}^i_c\left(s_i,s_{(3-i)}\right)\right).$$

Although this formulation demonstrates how contracting in the presence of possible ex post disputes can be accommodated by a standard model with a common prior and expected utility, we find this unsatisfactory as it involves quantifying the possibility of dispute by a precise probability, and a precise utility value. In sum, our approach differs by dispensing both with a common prior and a precise utility value in the case of disputes. We will use our illustrative example in the remainder of the paper to show that the standard model cannot capture our intentions.

Since the possibility of a dispute stems from the limitations of the model of the world each individual has, it becomes even more problematic to understand where the 'common prior' comes from. The attractive feature of Gul and Pesendorfer's EUU model is that the prior reflects the precision of the individual's own model of the world *and no more*. That part which can be precisely modelled for the individual (events which are measurable with respect to her own interpretation of which contingencies have and have not occurred) are assigned a precise or crisp probability. However, those events which reflect the lack of understanding the individual has about certain aspects of the underlying structure of the world (reflected in her possibility of dispute sets) are not measurable with respect to her prior. We find this appropriate since it reflects the ambiguity she perceives there to be. Correspondingly her preferences over more general acts will depend on her attitude towards such ambiguity as is reflected in her interval utility function.

4.2. The illustrative example continued

Returning to our illustrative example, recall $S=\{\boldsymbol{W},\boldsymbol{B},\boldsymbol{R}\}$. The possibility of dispute relations are given by

$$\begin{aligned}\mathcal{D}^1 &= \left\{(\mathrm{W},\mathrm{W}),(\mathrm{B},\mathrm{W}),(\mathrm{B},\mathrm{B}),(\mathrm{R},\mathrm{R})\right\},\\ \mathcal{D}^2 &= \left\{(\mathrm{W},\mathrm{W}),(\mathrm{W},\mathrm{B}),(\mathrm{B},\mathrm{B}),(\mathrm{R},\mathrm{R})\right\}.\end{aligned}$$

Thus the possibility of disputes sets for player 1 and player 2 are given by:

$$\begin{aligned}\mathcal{D}^1(\mathbf{W}) &= \{\mathbf{W}\},\ \mathcal{D}^1(\mathbf{B})=\{\mathbf{B},\mathbf{W}\},\mathcal{D}^1(\mathbf{R})=\{\mathbf{R}\},\\ \mathcal{D}^2(\mathbf{W}) &= \{\mathrm{W},\mathrm{B}\},\ \mathcal{D}^2(\mathbf{B})=\{\mathbf{B}\},\mathcal{D}^2(\mathbf{R})=\{\mathbf{R}\}.\end{aligned}$$

The prior $\left(\mathcal{E}^1,\mu^1\right)$ on $S^1\times S^2$ is admissible if $\mathcal{E}^1$ is the algebra generated from the partition:

$$\left\{\{(\mathrm{W},\mathrm{W})\},\{(\mathrm{W},\mathrm{B}),(\mathrm{W},\mathrm{R})\},\{(\mathrm{B},\mathrm{W}),(\mathrm{B},\mathrm{B})\},\{(\mathrm{B},\mathrm{R})\},\{(\mathrm{R},\mathrm{R})\},\{(\mathrm{R},\mathrm{W}),(\mathrm{R},\mathrm{B})\}\right\}$$

and

$$\mu^1(\{(\mathrm{W},\mathrm{B}),(\mathrm{W},\mathrm{R})\})=\mu^1(\{(\mathrm{B},\mathrm{R})\})=\mu^1(\{(\mathrm{R},\mathrm{W}),(\mathrm{R},\mathrm{B})\})=0.$$

Similarly, the prior $\left(\mathcal{E}^2,\mu^2\right)$ on $S^2\times S^1$ is admissible if $\mathcal{E}^2$ is the algebra generated from the partition:

$$\left\{\{(\mathrm{W},\mathrm{W}),(\mathrm{W},\mathrm{B})\},\{(\mathrm{W},\mathrm{R})\},\{(\mathrm{B},\mathrm{B})\},\{(\mathrm{B},\mathrm{W}),(\mathrm{B},\mathrm{R})\},\{(\mathrm{R},\mathrm{R})\},\{(\mathrm{R},\mathrm{W}),(\mathrm{R},\mathrm{B})\}\right\}$$

and

$$\mu^2(\{(\mathrm{W},\mathrm{R})\})=\mu^2(\{(\mathrm{B},\mathrm{W}),(\mathrm{B},\mathrm{R})\})=\mu^2(\{(\mathrm{R},\mathrm{W}),(\mathrm{R},\mathrm{B})\})=0.$$

For concreteness, further suppose that

$$\mu^1(\{(\mathrm{W},\mathrm{W})\})=\mu^1(\{(\mathrm{B},\mathrm{W}),(\mathrm{B},\mathrm{B})\})=\mu^1(\{(\mathrm{R},\mathrm{R})\})=\frac{1}{3};\qquad(2)$$

$$\mu^2(\{(\mathrm{W},\mathrm{W}),(\mathrm{W},\mathrm{B})\})=\mu^2(\{(\mathrm{B},\mathrm{B})\})=\mu^2(\{(\mathrm{R},\mathrm{R})\})=\frac{1}{3}.\qquad(2)$$

We shall also assume that for both individuals, $v(\cdot)$ is a common continuous, strictly concave and strictly increasing utility function over final wealth.

Finally if we take the ambiguity attitude parameter $\alpha^i=\alpha>0$ to be the same for both players, then the expected uncertain-utility of a contract for player 1 and player 2, are generated, respectively, by the functionals:

$$\begin{aligned}V^1(c) &= \frac{1}{3}v(c(\mathbf{W})+2)+\frac{1}{3}\left[(1-\alpha)\max_{s\in\{\mathbf{B},\mathbf{W}\}}v(c(s)+1)+\alpha\min_{s\in\{\mathbf{B},\mathbf{W}\}}v(c(s)+1)\right]\\ &\quad+\frac{1}{3}v(c(\mathbf{R})+3)\end{aligned}\qquad(3)$$

$$\begin{aligned} V^2(c) &= \frac{1}{3}\left[(1-\alpha)\max_{s\in\{\mathbf{B},\mathbf{W}\}} v(-c(s)+2)+\alpha \min_{s\in\{\mathbf{B},\mathbf{W}\}} u(-c(s)+2)\right] \\ &+\frac{1}{3}v(-c(\mathbf{B})+3)+\frac{1}{3}v(-c(\mathbf{R})+1). \end{aligned} \tag{4}$$

Some aspects of these preferences are noteworthy. The players' (*ex ante*) preference for signing a given hedging contract will be stronger the more risk-averse they are, that is, the stronger their preference for the non-stochastic allocation over the original endowment. Their preference for signing a hedging contract will be less the more weight they place on the possibility of different interpretations giving rise to disputes. Thus risk and ambiguity work in opposite directions. This result applies generally to problems involving ambiguous risk sharing contracts.

Before discussing the bargaining problem we mention some limitations with the standard state space approach discussed in the previous subsection. Let's think about the common prior μ for the commonly known state space $S^1 \times S^2$. First observe that for consistency with the common prior, from 1's perspective, we should have $\mu^1(\{(\mathrm{W},\mathrm{W})\}) = \mu(\mathrm{W},\mathrm{W}) + \mu(\mathrm{W},\mathrm{B})$. In our description, the case where 1 sees white and 2 sees black does not occur, so $\mu(\mathrm{W},\mathrm{B}) = 0$. By (2) we have that that $\mu(\mathrm{W},\mathrm{W}) = 1/3$. Next, considering 2's perspective, we should have $\mu^2(\{(\mathrm{W},\mathrm{W}),(\mathrm{W},\mathrm{B})\}) = \mu(\mathrm{W},\mathrm{W}) + \mu(\mathrm{B},\mathrm{W})$. However, by (2) we have that $\mu(\mathrm{B},\mathrm{W}) = 0$. In sum, the common prior assumption in our illustrative example requires that disputes arise with zero probability. Hence, the problem of disputes cannot be discussed.

5. The bargaining problem

We consider now the bargaining the two individuals can engage in, where the set of alternatives over which bargaining is to be conducted is taken to be some subset of the set of contracts C, characterized in Section 3. We assume that there is a designated contract $c_0 \in C$, which we take to be the disagreement action that will result should the bargaining process break down and no agreement is reached. We restrict our attention in what follows to individuals with EUU preferences that admit a representation of the form (1).

As C and c_0 will be fixed throughout, we shall identify the bargaining problem with the pair of preferences relations of the bargainers over C. Thus a bargaining problem in our set-up can be identified by a tuple $\left(\left\langle \mathcal{E}^1,\mu^1,v^1,\alpha^1\right\rangle,\left\langle \mathcal{E}^2,\mu^2,v^2,\alpha^2\right\rangle\right)$ with associated utility representations V^1 and V^2. So that the problem is not vacuous, we assume there exists a contract $\hat{c}$ in C such that $V^i\left(\hat{c}\right) > V^i(c_0)$, $i = 1, 2$. That is, $\hat{c}$ (strictly) Pareto dominates c_0.

Denote by $\mathcal{B}$ the class of bargaining problems for the analysis. To aid the analysis, it is convenient to define the *cardinal bargaining problem* induced by the preferences of the two bargainers in the following way.

Definition 4. Fix a bargaining problem $\left(\left\langle \mathcal{E}^1,\mu^1,v^1,\alpha^1\right\rangle,\left\langle \mathcal{E}^2,\mu^2,v^2,\alpha^2\right\rangle\right)$ in $\mathcal{B}$. The *cardinal bargaining problem* associated with this bargaining problem is the set $B \subset \mathbb{R}_2$, given by

$$B = \left\{(v_1,v_2) : \exists c \in C, (v_1,v_2) \le \left(V^1(c),V^2(c)\right)\right\}.$$

Notice that B is comprehensive by construction. Since A_0 is compact, it follows that C is compact as well, and hence it follows by the construction of B that it is closed. To allow for a simple and convenient characterization of the set of individually rational and efficient contracts in a given bargaining problem we assume the bargaining problem exhibits the following property introduced by Grant and Kajii (1995).

Definition 5 *(C-Convexity).* A bargaining problem $\left(\left\langle \mathcal{E}^1,\mu^1,v^1,\alpha^1\right\rangle,\left\langle \mathcal{E}^2,\mu^2,v^2,\alpha^2\right\rangle\right)$ in $\mathcal{B}$ exhibits *C-convexity* if for any pair of contracts c and c' in C, there exists a contract c'' in C such that

$$V^i\left(c''\right) \ge \frac{1}{2}V^i(c) + \frac{1}{2}V^i\left(c'\right),\ i = 1, 2.$$

A sufficient condition for a bargaining problem to exhibit C-convexity, is for the (state-dependent) utility functions $v^i \circ \left(y^i(\cdot,\sigma)\right) : A \to \mathbb{R}$ to be concave in a.[8]

As the name suggests, a bargaining problem that exhibits C-convexity has associated with it a convex cardinal bargaining problem.

Lemma 2. *If* $\left(\left\langle \mathcal{E}^1,\mu^1,v^1,\alpha^1\right\rangle,\left\langle \mathcal{E}^2,\mu^2,v^2,\alpha^2\right\rangle\right)$ *in* $\mathcal{B}$ *is a C-convex bargaining problem then the associated cardinal bargaining problem B is convex.*

Proof. Fix an arbitrary pair (v_1, v_2) and $\left(v_1', v_2'\right)$ in B. To establish that B is convex it is sufficient to show that $\left((1/2)v_1 + (1/2)v_1', (1/2)v_2 + (1/2)v_2'\right)$ is also in B. Since (v_1, v_2) and $\left(v_1', v_2'\right)$ are both in B, it follows from the definition

8 This holds naturally for risk-sharing contracts in which the action $a \in A_0 \subset \mathbb{R}$ corresponds to a transfer of size a from bargainer 2 to bargainer 1, and $v^i \circ \left(y^i(\cdot,\sigma)\right) = \mu^i_{\Sigma^i}(\sigma)v\left(a \times (-1)^{i-1} + z^i_s\right)$ is the probability weighted utility of bargainer i's final wealth in state s after the transfer has been made. Concavity of $v^i \circ \left(y^i(\cdot,\sigma)\right)$ then follows naturally from risk aversion (that is, concavity of v the utility index over wealth).

of B that there exists contracts c and c' in C, such that $(V^1(c), V^2(c)) \geq (v_1, v_2)$ and $(V^1(c'), V^2(c')) \geq (v'_1, v'_2)$. By C-convexity there exists a contract c'' in C, such that $V^i(c'') \geq (1/2)V^i(c) + (1/2)V^i(c')$, $i = 1, 2$. Hence,

$$\begin{aligned} \left(V^1(c''), V^2(c'')\right) &\geq \left(\frac{1}{2}V^1(c) + \frac{1}{2}V^1\left(c'\right), \frac{1}{2}V^2(c) + \frac{1}{2}V^2\left(c'\right)\right) \\ &\geq \left(\frac{1}{2}v_1 + \frac{1}{2}v'_1, \frac{1}{2}v_2 + \frac{1}{2}v'_2\right), \end{aligned}$$

as required. □

If all bargaining problems in $\mathcal{B}$ are convex, then for each problem we have a simple characterization of the set of individually rational and efficient contracts: they are the contracts that (a) are at least as good for both individuals as the disagreement contract c_0 and (b) maximize a weighted utilitarian social welfare function, for some set of (normalized) non-negative weights. Observe that although we have expanded $\mathcal{B}$ to include utility vectors that may not correspond to a contract c in C, the efficient vectors in $\mathcal{B}$ always do. Thus, we have the following proposition.

Proposition 3. *Suppose the bargaining problem* $\left(\left\langle \mathcal{E}^1, \mu^1, v^1, \alpha^1 \right\rangle, \left\langle \mathcal{E}^2, \mu^2, v^2, \alpha^2 \right\rangle\right)$ *in* $\mathcal{B}$ *is C-convex. Then the contract* c^* *is individually rational and efficient if and only if*

$$\min\left\{\left[V^1(c^*) - V^1(c_0)\right], \left[V^2(c^*) - V^2(c_0)\right]\right\} \geq 0$$

and

$$c^* \in \underset{c \in C}{\operatorname{argmax}} \lambda V^1(c) + (1-\lambda)V^2(c), \text{ for some } \lambda \text{ in } [0, 1]. \tag{5}$$

The proof of Proposition 3 follows from the application of a standard separating hyperplane theorem for convex sets and so is omitted.

5.1. The illustrative example continued

Taking $c_0 = 0$ (that is, no transfer is made), and denoting $V^1(c_0) = V^2(c_0) = \overline{u}$, an individually rational and efficient contract c for the illustrative example developed in Section 4.2 is one for which $V^1(c) \geq \overline{u}$, $V^2(c) \geq \overline{u}$ and c is a solution to the maximization problem,

$$\max_{\left\langle (c(\mathbf{W}), c(\mathbf{B}), c(\mathbf{R})) \in [-3,3]^3 \right\rangle} \lambda V^1(c) + (1-\lambda)V^2(c), \text{ for some} \lambda \text{ in } [0, 1].$$

For a given λ in [0, 1]the solution c^*_λ with associated contingent transfers $c^*_\lambda(\mathbf{R}) \leq c^*_\lambda(\mathbf{W}) \leq c^*_\lambda(\mathbf{B})$, satisfies the first-order conditions:

$$\begin{aligned} c^*_\lambda(\mathbf{W}) &: \lambda\left[v'\left(c^*_\lambda(\mathbf{W}) + 2\right) + \alpha v'\left(c^*_\lambda(\mathbf{W}) + 1\right)\right] - (1-\lambda)\left[(1-\alpha)v'\left(-c^*_\lambda(\mathbf{W}) + 2\right)\right] = 0. \\ c^*_\lambda(\mathbf{B}) &: \lambda(1-\alpha)v'\left(c^*_\lambda(\mathbf{B}) + 1\right) - (1-\lambda)\left[\alpha v'\left(-c^*_\lambda(\mathbf{B}) + 1\right) + v'\left(-c^*_\lambda(\mathbf{B}) + 3\right)\right] = 0 \\ c^*_\lambda(\mathbf{R}) &: \lambda v'\left(c^*_\lambda(\mathbf{R}) + 3\right) - (1-\lambda)v'\left(-c^*_\lambda(\mathbf{R}) + 1\right) = 0. \end{aligned}$$

Provided λ is in (0, 1), rearranging, we obtain:

$$\begin{aligned} &\frac{v'\left(c^*_\lambda(\mathbf{W}) + 2\right) + \alpha v'\left(c^*_\lambda(\mathbf{W}) + 1\right)}{(1-\alpha)v'\left(-c^*_\lambda(\mathbf{W}) + 2\right)} \\ &= \frac{(1-\alpha)v'\left(c^*_\lambda(\mathbf{B}) + 1\right)}{v'\left(-c^*_\lambda(\mathbf{B}) + 3\right) + \alpha v'\left(-c^*_\lambda(\mathbf{B}) + 2\right)} = \frac{v'\left(c^*_\lambda(\mathbf{R}) + 3\right)}{v'\left(-c^*_\lambda(\mathbf{R}) + 1\right)} = \frac{(1-\lambda)}{\lambda}. \end{aligned} \tag{6}$$

For the symmetric weighted utilitarian social welfare function (that is, $\lambda = 1/2$), we see immediately from (6) that for $\alpha = 0$, the solution is $(0, 1, -1)$. That is, when the decision maker places all the weight from the possibility of dispute on his own interpretation being implemented, the symmetric solution is the full risk-sharing contract described in Section 2.

To see what happens for $\lambda = 1/2$ and $\alpha > 0$, we have from (6) that $c^*_{1/2}(\mathbf{R}) = 1$ and furthermore whenever $c^*_{1/2}(\mathbf{B}) > 1/2 > c^*_{1/2}(\mathbf{W})$ holds, $c^*_{1/2}(\mathbf{B})$ and $c^*_{1/2}(\mathbf{W})$ are the unique solutions to:

$$\frac{1}{(1-\alpha)} \frac{v'\left(-c^*_{1/2}(\mathbf{B}) + 3\right)}{v'\left(c^*_{1/2}(\mathbf{B}) + 1\right)} + \frac{\alpha}{(1-\alpha)} \frac{v'\left(-c^*_{1/2}(\mathbf{B}) + 2\right)}{v'\left(c^*_{1/2}(\mathbf{B}) + 1\right)} = 1 \tag{7}$$

$$\frac{1}{(1-\alpha)}\frac{v'\left(c^*_{1/2}(\mathbf{W})+2\right)}{v'\left(-c^*_{1/2}(\mathbf{W})+2\right)}+\frac{\alpha}{(1-\alpha)}\frac{v'\left(c^*_{1/2}(\mathbf{W})+1\right)}{v'\left(-c^*_{1/2}(\mathbf{W})+2\right)}=1 \quad (8)$$

respectively. Notice that the LHS of (7) is increasing in $c^*_{1/2}(\mathbf{B})$ and the LHS of (8) is decreasing in $c^*_{1/2}(\mathbf{W})$, so while $c^*_{1/2}(\mathbf{B}) > (1/2) > c^*_{1/2}(\mathbf{W})$ the solution is well-defined for each corresponding α. Moreover, the payment $c^*_{1/2}(\mathbf{W})$ is strictly increasing, while the payment $c^*_{1/2}(\mathbf{B})$ is strictly decreasing, in α up to some critical value $\hat{\alpha} = [1 - (v'(5/2)/v'(3/2))]/2$ where those transfers equal 1/2.

This can be interpreted as follows. Recall that as α increases, each party places more weight on the worst outcome, i.e., ambiguity has more bite. This motivates the parties to move closer to the optimal unambiguous contract. In the limiting case where $\alpha = \hat{\alpha}$, the optimal contract is the unambiguous contract with contingent transfers $(1/2, 1/2, -1)$, that is, if the card drawn is red then player 1 pays 1 to player 2, and otherwise player 2 pays 1/2 to player 1. This remains the optimal contract for any $\alpha > \hat{\alpha}$, since the contingent transfers $(1/2, 1/2, -1)$ satisfy

$$\frac{v'(c^*_{1/2}(\mathbf{R})+3)}{v'(-c^*_{1/2}(\mathbf{R})+1)}=\frac{v'(c^*_{1/2}(\mathbf{B})+1)+v'(c^*_{1/2}(\mathbf{W})+2)}{v'(-c^*_{1/2}(\mathbf{B})+3)+v'(-c^*_{1/2}(\mathbf{W})+2)}=1,$$

the first-order conditions for the optimal *unambiguous* contract.

If we apply the standard state space approach to this example, then as mentioned in the previous section, the dispute outcome has zero probability of occurrence. Hence, for $\lambda = 1/2$, the optimal contract is $(0, 1, -1)$, which happens to coincide with our model only in the case where both parties have $\alpha_i = 0$.

6. Concluding comments

We have provided a formal model for incorporating ambiguity into decision making. The ambiguity in our model arises from the bounded rationality of the players which is expressed as limited abilities to perform tests over the possible contingencies. This limitation results in each player having a limited individual description of the world.

The analysis presented here has been undertaken entirely in semantic terms, with no explicit description of the language in which contracts are written. An alternative approach, explored in a companion paper (Grant et al., 2011), would be to begin with an explicit syntactic representation of the set of possible contracts, and derive the state space in a manner similar to that of Blume et al. (2006). This approach provides some additional insights, particularly regarding the nature of ambiguity. However, the relationship between syntactic and semantic formulations of contracting problems requires further exploration, which is beyond the scope of this paper.

The representation of ambiguity proposed here suggests new approaches to a range of issues in contract theory. Some of these issues have proved difficult to address using approaches based on unbounded rationality, or on arbitrary constraints on rationality. In the case of risk sharing, we have shown that ambiguity may lead players to prefer incomplete risk sharing to possibly ambiguous contracts.

The analysis suggests the possibility of broader applications in agency theory. The standard principal–agent problem is one where contracting is limited to some observable unambiguous characteristics like output, rather than a full set of characteristics including effort levels which may be ambiguous. The framework developed here suggests the possibility of an endogenous choice between contracts over different characteristics, where the choice of the contractual variables chosen depends on the level of ambiguity and potential gains from risk sharing. While this application would require overcoming some new technical details involving the appropriate treatment of tests, the benefit would be the development of a theory of contracting in which the terms of the contract, over which the parties actually bargain, plays the central role.

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