The Risk Premium for Equity: Implications for the Proposed Diversification of the Social Security Fund

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The United States Social Security system has accumulated unfunded liabilities estimated at $9 trillion (John Geanakoplos et al., 1998). The need to meet these liabilities implies, other things being equal, a need for higher taxes in the future. The need for transitional policies to fund the accumulated liability has generated increased interest in proposals for policy change which may yield improvements in efficiency and, in particular, improvements in the return to Social Security investments. A number of proposals addressing the unfunded liabilities involve dropping the requirement that the assets of the Social Security fund should be invested solely in bonds and allowing some of the assets of the fund to be invested in equity. Critics such as Alan Greenspan (1999) have observed potential conflicts of interest associated with public ownership of equity.

A more fundamental criticism has been the observation that, in the absence of capital-market imperfections or restrictions on the capacity of individuals to diversify risk, the diversification of Social Security investments into stocks will be offset by reallocation of individual asset portfolios. More precisely, as Geanakoplos et al. (1998) show, under the assumptions of optimization, time homogeneity, stable prices, and spanning, the diversification of Social Security investments into stocks has no effect on measures of the "money's worth" of Social Security. This result holds whether diversification is achieved through privatization or through a change in the investment policy of the Social Security fund.

The initial attractiveness of proposals to diversify Social Security investments arises primarily from the large difference between the average rates of return to bonds and equity, referred to as the equity premium. As Rajnish Mehra and Edward C. Prescott (1985) observe, the magnitude of the equity premium is a theoretical puzzle. In most models of asset-price determination based on the assumption that individuals operating in efficient capital markets rationally optimize consumption over time, the equilibrium returns to equity and bonds differ by less than 1 percent.

A number of writers have argued that the anomalous behavior of asset prices reflects capital-market imperfections. Two main types of imperfections have been considered. First, imperfect risk-spreading within generations may arise from moral hazard and adverse selection problems (N. Gregory Mankiw, 1986). Second, imperfect consumption smoothing may arise from borrowing constraints or transactions costs which restrict trade between generations (John Heaton and Deborah J. Lucas, 1996; George M. Constantinides et al., 1998).

Relatively little attention has been paid to the policy implications of the equity-premium and risk-free rate puzzles. However, as we have observed (Grant and Quiggin, 1999), the welfare effects of public investments depend crucially on the analysis of asset-price determination. If the equity premium arises from adverse selection problems, which prevent risk-spreading through market transactions, the tax system (which is not subject to adverse selection) provides a potentially superior method of risk-spreading.[1]

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[1] Similar benefits may arise if some individuals are constrained from saving, as in Peter Diamond and Geanakoplos (1999).

Very similar issues arise in assessing the proposal to reallocate Social Security investments from bonds to equity. Suppose that asset prices are determined in perfectly efficient markets and that taxpayers treat risk about net tax liabilities in the same way as they treat risk about income from direct ownership of capital.[2] They will therefore regard themselves as owning a share in any publicly owned assets. A reallocation of the public portfolio which does not affect the distribution of income will lead to an offsetting reallocation of privately held assets which, under appropriate conditions, will leave equilibrium asset prices unchanged.[3] If, on the other hand, the equity-premium and risk-free rate puzzles arise from capital-market imperfections then it is possible that public sales of bonds and purchases of equity may have the effect of raising the return to bonds and reducing returns to equity, and that these changes may increase welfare.

As noted above, the welfare benefits of diversification of Social Security investments depend ultimately on the capacity of government to spread risk through the tax system. It is therefore important to consider whether a proposal for diversification may be interpreted simply as a welfare-increasing tax reform combined with an unrelated proposal for government purchases of equity.

The object of this paper is to examine these issues in a simple two-period model, which permits the derivation of an analytical solution to the problem of determining equilibrium asset prices in the presence of undiversifiable risk associated with adverse selection problems. The approach is, therefore, similar to that of Mankiw (1986) and Philippe Weil (1992). Our innovation is to introduce a government with the power to levy a proportional labor income tax and an obligation to make a specific defined payment in the second period. We also allow government investment in equity and compare the effects of such investments with and without complete risk pooling in private capital markets. Assuming that agents exhibit decreasing absolute risk aversion, we show that, in the absence of private risk pooling, public ownership of equity will improve welfare. Decreasing absolute risk aversion means that, in utility terms, the loss from a given increase in risk is greater at lower levels of income (see Josef Hadar and William R. Russell, 1969). Hence, *ex ante* welfare is increased by a policy that increases risk when income is high and reduces risk when income is low.

The proposal for purchase of equity is then compared with a tax reform proposal not involving purchase of equity, based on that of Robert B. Barsky et al. (1986). In their proposal, second-period taxes are used to repay debt generated by a first-period budget deficit. It is shown that, particularly when the elasticity of labor supply is taken into account, the diversification proposal is *ex ante*. Pareto superior to that of Barsky et al.

I. Generations

The formal analysis presented in this paper employs a simple two-period model, since such a model enables us to analyze the critical issues without distracting complications. However, the two-period model considered here may usefully be regarded as a subset of an overlapping-generations model, with three generations: young, middle-aged, and old. Unlike most overlapping-generations models, where attention is focused on dynamically stable equilibria with fixed institutions, we consider a transition from one set of institutions to another. As Geanakoplos et al. (1998) emphasize, it is the unavoidable *transitional* cost that is crucial in understanding the problems of the Social Security fund.

We assume that for some time prior to the present, retirement income has been provided through a social security scheme, under which the young and middle-aged pay taxes to finance defined benefits received in old age. The scheme is not self-funding, that is, the present value of net benefits received by any given cohort is positive. However, until the present period, denoted as period 1, income growth has been such that the scheme is sustainable with a fixed level of taxation. Looking ahead to period 2, it is evident that taxes will have to be raised to meet the obligation to those who will be old

[2] This equivalence is similar to the equivalence between individual and corporate debt required for the Modigliani-Miller "homemade leverage" proposition to hold. When viewed in an intertemporal context, the rationality requirement is similar to that needed for Ricardian equivalence.

[3] This neutrality property may not hold in an economy with endogeneous growth. Andrew B. Abel (1999) shows in this case that the reallocation of Social Security funds to equity will reduce the equity premium and may increase the growth rate of capital along a constant growth path.

in that period. The scheme will be scrapped (or privatized) so that no benefits will be payable after period 2, so we focus our attention on the question of financing this once-off liability payable in period 2.

As period 1 retirees (passively) consume the Social Security payments that are paid out in period 1, their consumption will not be explicitly modeled. Furthermore, we shall assume that the middle-aged workers in period 1, who will be retirees in period 2 and who will be the beneficiaries of the Social Security payments paid out in period 2, consume all their disposable income in period 1. Hence they can also be "netted out" from the formal analysis as their consumption in both periods is also predetermined. Finally, we assume that any contribution made by the generation who are young in period 2 can be netted out.

These simplifications allow us to focus our attention on those who are young in period 1 and will be middle-aged in period 2. They must decide how to meet the once-off obligation to pay benefits in period 2. The crucial issue is whether the government can improve the welfare of the young today by acquiring equity in period 1 to assist in financing its obligation to meet Social Security payments to the old in period 2. Once the return to this investment is realized in period 2, the necessary labor income tax rate is determined by the difference between investment income and the benefit liability.

II. The Model

In order to keep the analysis as transparent as possible and to avoid having to track distributions of consumption, we follow Weil (1992) and introduce a two-period Robert E. Lucas, Jr. (1978) style economy, in which there are a continuum of *ex ante* identical (young) workers defined over the interval [0, 1]. They are assumed to be expected utility maximizers having tastes over consumption and leisure represented by additively separable utility preferences of the following form:

$$(1) \qquad v_t(c_t) - h_t(\ell_t) \qquad t = 1, 2, 3$$

where $v_t(\cdot)$ is a strictly increasing and strictly concave utility function, ℓ_t is labor supply in period t and $h_t(\cdot)$ is a strictly increasing and strictly convex disutility of labor function. Noting that ℓ_3 is identically zero, we can combine the second and third periods to yield preferences of the form

$$(2) \qquad u_t(c_t) - h_t(\ell_t) \qquad t = 1, 2$$

where $u_1(\cdot) = v_1(\cdot)$ is the utility function defined over period-1 consumption c_1, c_2 denotes second-period wealth, and $u_2(\cdot)$ is the indirect utility function defined over second-period wealth. Both $u_1(\cdot)$ and $u_2(\cdot)$ are strictly increasing and strictly concave. We assume that $u_t(\cdot)$ displays relative risk aversion less than 1, so that the labor-supply curve is upward-sloping in each period.

For each i in [0, 1], consumer i receives a pretax labor income

$$y_1 = w_1 \ell_1$$

in the first period, where w_1 is the period 1 wage.

In period 2, the wage for each i is a *random* variable W_i.[4] Moreover, the supply of labor for some individuals may be constrained because of unemployment. Hence, the individual's pretax wage income is given by

$$Y_i = W_i L_i \qquad L_i \leq \bar{L}_i.$$

Two polar cases are considered. In the *labor-market clearing case*, there are no unemployment constraints. Random variation in Y_i arises solely from variation in W_i and the resulting endogenous labor-supply response. In the *Keynesian involuntary unemployment case*, the wage is nonstochastic and variation in second-period income arises solely from the unemployment constraint.

In addition to their endowment of labor hours, all young workers are endowed at birth with the same number (normalized to 1) of shares of a two-period lived tradable asset that we shall refer to as "equity." The dividend, payable in the second period, D, is random. Workers may also buy and sell a risk-free bond which pays unconditionally one unit of the

[4] Throughout, capital letters will denote random variables (that is, real functions defined on the underlying state space) and lowercase letters will denote realizations and nonrandom variables. Since there is no uncertainty in period 1, we suppress the time subscript for random variables.

consumption good in period 2. All workers are endowed with zero units of the risk-free bond. Adverse selection problems, modeled in more detail in Grant and Quiggin (1999), prevent workers from insuring themselves against risk in their second-period labor income. Thus, workers are faced with nondiversifiable idiosyncratic risk.[5]

The government is committed to providing in each of the two periods an amount s_t of Social Security payments to the retirees in that period. We assume there is only one tax instrument, a proportional labor income tax, and that the government sets first-period taxes at a level just sufficient to meet the Social Security obligation in that period,[6] so with an appropriate normalization,

$$\tau_1 y_1 = s_1$$

where τ_1 is the labor income tax rate in period t. In the first period the government can also issue bonds and purchase equity. In the second period it supplements the (net) revenue derived from its first-period portfolio holding with a proportional labor income tax on second-period workers to meet any shortfall in covering its commitment to pay retirees s_2. This tax is levied at a rate T, which is, in general, a random variable. The transitional problem of financing the accumulated deficit is reflected in the assumption that $T > \tau_1$ with probability 1.

Let p and q denote, respectively, the prices of equity and bonds. Let (g^e, g^b) [respectively, (x_i, b_i)] denote the government's (respectively, worker i's) portfolio holding of equity and bonds in the first period. And let T denote the government's (state contingent) proportional income tax rate in the second period. The government's budget portfolio constraint in the first period can be expressed as:[7]

$$pg^e + qg^b = 0 \tag{3}$$

so that

$$\tau_1 y_1 = s_1 + pg^e + qg^b$$

and the government's (state-contingent) budget constraint in the second period is given by

$$T\bar{Y} = s_2 - Dg^e - g^b \tag{4}$$

where for each state ω, $\bar{Y}(\omega) = \int Y_i(\omega)\, di$ is the (state-contingent) per capita level of labor income.

Similarly, each worker i faces in the first period the budget constraint:

$$c_{1i} + px_i + qb_i = p + (1 - \tau_1)y_1 c_{1i} \geq 0. \tag{5}$$

Along with her state-contingent second-period labor income, Y_i, and the state-contingent labor income tax rate, T, that satisfies (4), her portfolio choice (x_i, b_i) in the first period leads to a second-period random wealth of

$$C_i = (1 - T)Y_i + Dx_i + b_i. \tag{6}$$

Since (young) workers are risk-averse and identical *ex ante* (although not *ex post*) they will not trade with each other in equilibrium. Hence the characterization of the (rational expectations) equilibrium simply involves finding asset prices that support the consumers' initial endowment less the government's portfolio choice (g^e, g^b). Hence the equilibrium holdings of equity and bonds for each worker i must be

$$x_i = \bar{x} = 1 - g^e \tag{7}$$

where $\bar{x} = \int x_j\, dj$ (per capita holding of equity) and

$$b_i = \bar{b} = -g^b \tag{8}$$

[5] Christian Gollier and John W. Pratt (1996) discuss comparative statics of choice in the presence of nondiversifiable background risk and note the relevance of their analysis to the analysis of the equity premium.

[6] We relax the first assumption later in considering the Barsky et al. (1986) proposal.

[7] For analytical convenience we have taken the value of the government's net position to be zero, but this is without any essential loss of generality. Qualitatively the results we derive would still hold if the government were "endowed" with an outstanding stock of debt (which would have to be serviced in the second period) and it had a "surplus" from the labor income tax in period 1 which more than covered the government's Social Security payments for this period. In this case the issue would be how much of the surplus should be used to reduce the outstanding stock of debt (i.e., effectively "investing" the tax surplus in bonds) versus using the surplus to purchase equity.

where $\bar{b} = \int b_j\,dj$ (per capita holding of bonds).

Combining (7) and (8) with the government's portfolio constraint and the consumer's first-period budget constraint [i.e., (3) and (5)] yields

$$c_{1i} = y_1(1 - \tau_1).$$

Noting that, for an interior solution,

$$w_1(1 - \tau_1)u_1'(y_1) = h_1'(\ell_1)$$

and taking the normalization $h_1'(\ell_1) = 1$, the equilibrium prices for equity and bonds may be expressed as the first-order conditions for the optimum holdings of equity and bonds, where for each i in [0, 1]

$$p = w_1(1 - \tau_1)\mathrm{E}[Du_2'(C_i)] \tag{9}$$

$$q = w_1(1 - \tau_1)\mathrm{E}[u_2'(C_i)] \tag{10}$$

and where E is the mathematical expectations operator.

Letting R_e (respectively, R_b) denote the (gross) return to holding equity (respectively, a bond) it readily follows from (9) and (10) that

$$\mathrm{E}[R_e] = \mathrm{E}\left[\frac{D}{p}\right] = \frac{\mathrm{E}[D]}{w_1(1 - \tau_1)\mathrm{E}[Du_2'(C_i)]}$$

$$\mathrm{E}[R_b] = \mathrm{E}\left[\frac{1}{q}\right] = \frac{1}{w_1(1 - \tau_1)\mathrm{E}[u_2'(C_i)]}$$

and thus, the equilibrium equity premium in ratio form, denoted by π, may be expressed as

$$\begin{aligned} \pi \equiv \frac{\mathrm{E}[R_e]}{\mathrm{E}[R_b]} &= \frac{\mathrm{E}[D]/p}{1/q} \\ &= \frac{\mathrm{E}[D]\mathrm{E}[u_2'(C_i)]}{\mathrm{E}[Du_2'(C_i)]} \\ &= 1 - \frac{\mathrm{Cov}[D, u_2'(C_i)]}{p}. \end{aligned} \tag{11}$$

In Weil's (1992) analysis, D and Y_i are assumed to be statistically independent which means that risk aversion (that is, $u_2'' < 0$) is sufficient to ensure that $\mathrm{Cov}[D, u_2'(C_i)] < 0$ and, hence, $\pi > 1$.

A. *Diversification and the Distribution of Consumption*

We now consider the impact of diversification on the distribution of consumption for given labor income Y_i.[8] To examine more closely the effect the government's holding of equity has on the period distribution of second-period consumption, notice that by substituting the market-clearing conditions for the bond and equity markets [i.e., (7) and (8)] and the government's portfolio constraint (3) into (6), the expression for an individual's second-period consumption, we obtain

$$C_i = (1 - T)Y_i + D(1 - g^e) + \frac{p}{q}g^e \tag{12}$$

and from the government's second-period budget constraint (4) and first-period portfolio constraint (3) we have

$$(1 - T) = \frac{\bar{Y} + (D - p/q)g^e - s}{\bar{Y}}. \tag{13}$$

Hence each worker i's random second-period consumption may be expressed as

$$\begin{aligned} C_i &= D + \bar{Y} - s \\ &\quad + \left(\frac{\bar{Y} + (D - p/q)g^e - s}{\bar{Y}}\right)(Y_i - \bar{Y}). \end{aligned} \tag{14}$$

Set $\bar{C} := \int C_i\,di$. $\bar{C}$ is the (state-contingent) per capita consumption of workers in the second period. From (14) we see that $\bar{C} = D + \bar{Y} - s$, that is, the per capita second-period consumption of workers equals the sum of the second-period per capita dividend and labor income less the government's committed payment to second-period retirees. It immediately follows that if there is no idiosyncratic component to their labor income (that is, $Y_i = \bar{Y}$), then $C_i = D + \bar{Y} - s$ is independent of the government's choice of g^e, which in turn implies that $p =$

[8] The effects of tax on labor supply and income are considered in the next section.

$w_1(1 - \tau_1)\mathrm{E}[Du_2'(D + \bar{Y} - s)]$ and $q = w_1(1 - \tau_1)\mathrm{E}[u_2'(D + \bar{Y} - s)]$ are also determined independently of g^e. That is, if workers only face "aggregate" uncertainty in the second period, then the government's first-period portfolio choice is *neutral*, in the sense that asset prices and second-period consumption are unaffected.

This neutrality breaks down, however, if workers face undiversifiable risk associated with their labor income. Results from the literature on the portfolio problem with one risky asset and one safe asset may be used to show that, as would be expected, an increase in government purchases of equity, financed by the sale of bonds, will increase the relative price of equity to bonds and thus will reduce the equity premium π.

Consider now the welfare effects. If we assume, as Weil (1992) does, that D and Y_i are statistically independent then $\bar{Y}$ is a degenerate random variable (i.e., it is constant across all states). To see what the effects of government holdings of equity might be under this assumption, consider (12) and (13) and observe that, for values of g^e between 0 and 1 the existence of a government holding of equity induces additional variation in posttax labor income, which is undesirable, ceteris paribus. However, notice that if dividend income D is less than p/q, the payout from the government's equity holding does not cover the amount it owes to its bondholders and so the government must set a labor income tax rate T greater than $s/\bar{Y}$. From (14) we see this in turn means that the variation of posttax labor income (and hence second-period consumption) across individuals is reduced in periods when dividend income is low. Conversely, in periods in which the dividend income is high (i.e., $D > p/q$) the variation of posttax labor income is *increased*. The change in the distribution of an individual's second-period consumption induced by the government's holding of equity cannot be simply ranked in terms of risk aversion. If, however, we assume that young workers display decreasing absolute risk aversion then we can establish (as is formally shown in Proposition 1 below) that a small government holding of equity is *ex ante* welfare-enhancing for young workers. Decreasing absolute risk aversion means that, in utility terms, the loss from a given increase in risk is greater at lower levels of income (See Hadar and Russell, 1969). Hence, *ex ante welfare* is increased by a policy that increases risk when income is high and reduces risk when income is low.

Before proceeding further, it is useful to observe that

$$\partial C_i/\partial g^e = \frac{(D - p/q)}{\bar{Y}}(Y_i - \bar{Y}) - \frac{g^e}{\bar{Y}}\left[\frac{\partial}{\partial g^e}\left(\frac{p}{q}\right)\right](Y_i - \bar{Y}).$$

For small values of g^e, the second term will be dominated by the first. But for large values of g^e, if the relative price p/q is increasing in g^e then the second term will imply that increases in g^e provide a second-degree stochastic improvement in the distribution of second-period consumption for the young. As noted above, provided $g^e < 1$, the decreasing absolute risk aversion implies that the equilibrium relative price of the risky equity to risk-free bond is increasing (and hence the equilibrium equity premium is decreasing) as g^e is increased.

PROPOSITION 1: *Assume D and Y_i are statistically independent and that second-period preferences display decreasing absolute risk aversion (that is, $u_2'(c) > 0$, $u_2''(c) < 0$, and $-u_2''(c)/u_2'(c)$ is monotonically decreasing). Then their ex ante welfare is an increasing function of g^e, the government holdings of equity.*

PROOF:
See Appendix.

The assumption that Y_i and D are independently distributed may seem too strong and not accord very well with the empirical record. What may be viewed as the opposite polar assumption about the state-contingent distribution of workers' second-period income appears in Mankiw (1986), in which a single measure of aggregate (or systemwide risk) is concentrated on a small proportion of the population. This can be incorporated, however, into Weil's framework with an individual facing both aggregate systemwide risk and a personal or idiosyncratic risk associated with his or her labor income, by the requirement that the distribution of labor income across the population improves in the sense of second-order "stochastic" dominance for higher values of the second-period dividend. More formally, the relaxation of independence that we have in mind may be expressed as follows, for all pairs of states, ω and ω', and any strictly increasing concave function, f:

$$(15)\quad \left[\int_0^1 [f(Y_i(\omega)) - f(Y_i(\omega'))]\, di\right] \times [D(\omega) - D(\omega')] \geq 0.$$

One may interpret (15) as saying that, for any concave function, f, the pair of random variables $\int_0^1 f(Y_i)\, di$ and D are *co-monotonic*.[9]

Weil's assumption that Y and D are statistically independent, may be viewed as the special case of (15) in which the distribution of labor income across the population is invariant to the realization of the second-period dividend, thus yielding for all pairs of states, ω and ω', $\int_0^1 [f(Y_i(\omega)) - f(Y_i(\omega'))]\, di = 0$.

In Mankiw's specific model with two aggregate events, recession and boom, the co-monotonicity between D and the distribution of labor income takes the special form

$$Y_i|D = \begin{cases} y_L - p\varepsilon & \text{with probability } (1-p) \text{ if } D = d_L \\ y_L + (1-p)\varepsilon & \text{with probability } p \text{ if } D = d_L \\ y_H & \text{with probability 1 if } D = d_H. \end{cases}$$

where $\varepsilon > 0$ and $y_H > y_L$ (H refers to quantities in the boom event, and L to quantities in the recession event). Thus, for each individual, the aggregate recession event is divided into two personally relevant events (recession with job loss) and (recession without job loss). In the boom event, $C_i = d_H + y_H - s$ which is independent of g^e, but, in the recession event, the small holding of equity by the government induces a reduction in the variability of C_i. Hence, the change induced by the government taking a small holding of equity represents an improvement in the sense of second-order stochastic dominance. Thus in Mankiw's model, strict concavity of u is sufficient to ensure that such a policy increases the *ex ante* utility of every worker i. But more generally we also have:

COROLLARY 2: *If young workers display standard risk aversion and* (15) *holds then*

$$\frac{d}{dg^e}\mathrm{E}[u(C_i)] > 0.$$

B. *Diversification, Budget Balance and Labor Supply*

The requirement for budget balance implies that variations in the return on the public sector holding of equity must be offset by variations in the labor income tax rate. Other things equal, state-contingent variations in the labor income tax rate will create welfare-reducing distortions in the labor-supply decision. We begin by considering this issue in the context of a labor-market clearing model, where variation in Y_i arises solely from variation in the posttax wage $(1 - T)W_i$ and the resulting endogenous labor-supply response.

The first-order condition for labor supply in the additively separable model is

$$W_i(1-T)u_2'(C_i) = h'(\ell_{2i})$$

where

$$C_i = W_i(1-T)\ell_{2i} + Dx_i + b_i.$$

Differentiating with respect to T yields

$$-W_i u_2'(C_i) + W_i(1-T)u_2''(C_i) \times \left[W_i(1-T)\frac{\partial \ell_{2i}}{\partial T} - W_i\ell_{2i}\right] = h''(\ell_{2i})\frac{\partial \ell_{2i}}{\partial T}.$$

Rearranging, we have

$$\left(h''(\ell_{2i}) + (1-T)^2 W_i \frac{-u_2''(C_i)}{u_2'(C_i)}\right)\frac{\partial \ell_{2i}}{\partial T} = \frac{u_2''(C_i)(1-T)Y_i}{u_2'(C_i)} - 1.$$

[9] Two random variables, X and Y, are said to be co-monotonic, if for any pair of states, ω and ω', $[X(\omega) - X(\omega')][Y(\omega) - Y(\omega')] \geq 0$.

Hence, if

$$\alpha = \frac{-u_2''(C_i)(1-T)Y_i}{u_2'(C_i)} < 1$$

then

$$\frac{\partial \ell_{2i}}{\partial T} < 0.$$

That is, individuals respond to an increase in taxation by reducing labor supply. Since total income in period 2 includes dividend and interest income in addition to posttax labor income $(1 - T)Y_i$, the elasticity α represents a coefficient of partial risk aversion and is less than the coefficient of relative risk aversion. Hence the requirement for the latter to be less than one ensures that $\partial \ell_{2i}/\partial T < 0$.

In the Weil case, individual variation in W_i and Y_i is uncorrelated with variations in investment returns D, and there is no aggregate uncertainty in Y. Hence, in the absence of public sector holdings of equity, the tax rate will be some constant $\tau_2 > \tau_1$. Using the standard Harberger approximation, the welfare loss associated with a given labor income tax τ may be approximated by

$$\Delta = 0.5 W_i \frac{\partial \ell_{2i}}{\partial t} \tau_2^2.$$

It follows that the marginal loss associated with varying the state-contingent tax rate around the initial constant level τ_2 may be approximated by

$$\Delta = \mathrm{E}\left[0.5 W_i \frac{\partial \ell_{2i}}{\partial T}(T^2 - \tau_2^2)\right]$$

or linearizing around τ_2

$$\Delta \approx 0.5 W_i \frac{\partial \ell_{2i}}{\partial t}(\mathrm{E}[T^2] - \tau_2^2).$$

In general, the sign of $(\mathrm{E}[T^2] - \tau_2^2)$ is ambiguous. The existence of an equity premium $\pi > 1$ implies that the expected return arising from debt-financed purchases of equity is positive. Hence, if $g^e > 0$, $\mathrm{E}[T] < \tau_2$. On the other hand, since T is a random variable, $\mathrm{E}[T^2] > (\mathrm{E}[T])^2$. However, in a neighborhood of $g^e = 0$, the mean effect must dominate. Hence, for small values of g^e, the conclusion that diversification will increase welfare is strengthened by consideration of labor-supply effects.

Now consider a labor-market-clearing economy where wage income and profits are positively correlated, as in a real-business-cycle version of Mankiw's model. Thus, even in the absence of government holdings of equity, the tax rate required to meet the Social Security obligation will vary inversely with the average wage. This variation will be increased by the taxes required to balance variations in dividend income from government holdings of equity. This effect generates a first-order welfare loss from labor-supply distortions even in a neighborhood of $g^e = 0$. Moreover, the labor-supply distortion will exacerbate the variability of consumption and will therefore offset the risk reduction associated with diversification. An approximate formula for the welfare loss associated with distorting taxation is

$$\Delta \approx 0.5 W_i \frac{\partial \ell_{2i}}{\partial t} \times \left[\mathrm{E}[T^2] + \alpha \operatorname{cov}\left(T, \frac{Y_i}{\bar{Y}}\right)\right]. \tag{16}$$

Since both the risk-reduction benefits and the labor-supply distortion costs of diversification are greater in the Mankiw case than in the Weil case, the relative benefits or costs of diversification cannot be ranked unambiguously in the absence of specific conditions on the model parameters.

Finally, consider an involuntary unemployment case, where the wage is nonstochastic and variation in second-period income arises solely from the unemployment constraint. For this case, it is natural to focus on a Mankiw-style model where unemployment constraints apply in the recession state and are borne by a small proportion of the population. Since those subject to a labor-supply constraint are not affected by the wage tax distortion, the welfare loss Δ in (16) is an expectation calculated only over the boom event and the event (recession, no job loss). However, it is the event (recession, job loss) which contributes most of the covariance between T and $Y_i/\bar{Y}$. Hence, the welfare loss associated with labor-supply distortions will be smaller in the Keynesian involuntary unemployment version of the Mankiw model than in the market-clearing real-business-cycle version.

In all cases, the balance between the risk-reducing effects of diversification and the welfare costs of labor supply will depend on the partial risk-aversion parameter:

$$\alpha = \frac{u''(c)(1 - T)Y_i}{u'(c)}.$$

The closer is α to 1, the greater the risk-reduction benefit and the smaller the labor-supply response to variations in posttax wages.

More importantly, the balance between risk-reduction and labor-supply distortion will depend on the nature of fluctuations in aggregate income. In an economy with Keynesian involuntary unemployment, where profits and labor income covary strongly and recessions are characterized by a failure of the labor market to clear, the benefits of risk reduction will be relatively large and the costs of labor-supply distortion relatively small. In an economy where labor markets always clear, variations in aggregate income reflect variations in factor productivity, and there is no necessary correlation between labor income and profits, the reverse will be true.

III. Tax Reform Without Diversification

The requirement for budget balance in the model presented above implies that any change in the public holding of assets must be matched by a change in tax policy. It is important, therefore, to consider the possibility that the beneficial effects attributed to diversification of public holding of assets arise simply because of the risk-reducing effects of taxation, and that similar benefits could be achieved by any policy which required second-period taxes to offset first-period policy decisions.

Barsky et al. (1986) (hereafter BMZ) show that beneficial risk-reduction can be achieved if second-period taxes are used to repay debt generated by a first-period budget deficit. This policy proposal is of particular interest in the present context, since it is similar to the Social Security reform proposed by George W. Bush, in which a proportion of current-period Social Security taxes would be returned to young workers, with no commensurate reduction in the benefits paid to older workers, and the cost being met by a reduction in the budget balance (see Paul Krugman [2001] for a discussion of this issue).

As Dean Croushore (1996) observes, the results derived by BMZ depend on the assumption that labor supply is perfectly inelastic. With elastic labor supply, the optimal first-period deficit and the welfare benefits of the policy are substantially reduced. In this section, we compare the BMZ proposal with the diversification policy under a range of assumptions regarding labor supply.[10]

In the absence of labor-supply response, a BMZ-style proposal clearly dominates the proposal for diversification. For the Weil case, the labor income tax rate under the BMZ-style proposal is nonstochastic and the individual tax burden is perfectly negatively correlated with the wage. For the Mankiw case, the BMZ-style proposal directly offsets idiosyncratic labor-income risk, though not the systematic risk in aggregate income. By contrast, the diversification proposal merely offsets an independent background risk.

This conclusion breaks down when labor-supply response is considered. As noted above, the existence of the Social Security obligation implies that $T > \tau_1$ with probability 1. The BMZ-style proposal involves a tax cut in period 1 and a tax increase in period 2, which exacerbates the intertemporal labor-supply distortion.[11] The welfare loss associated with this labor-supply distortion is first-order even when the change in tax rates is small. By contrast, diversification yields a positive expected return to government (because of the equity premium) and therefore a reduction in the expected period 2 tax rate $E[T]$.[12] Hence, the welfare benefits of the BMZ-style proposal are considerably less

[10] We thank a referee for drawing our attention to the similarities and differences between diversification and the BMZ proposal.

[11] The problem modeled in this paper is less favorable to a BMZ-style policy response because of the future liability. In the case considered by BMZ and Croushore, the status quo has $t_1 = t_2$.

[12] In the absence of market failure, this expected benefit would be fully offset by the welfare cost of publicly borne risk. This is not the case here because of the idiosyncratic labor-income risk borne by individuals in the private sector.

robust to labor-supply response than are those of diversification.

This argument applies to policies in which the only control variable is a proportional tax on labor income. If the government has access to a policy instrument permitting state-contingent lump-sum transfers, the first-best can always be obtained. In practice, as the difficulties encountered by proposals for poll taxes and negative income taxes have shown, no lump-sum instrument exists even if the instrument is not required to discriminate between individuals.

It may be useful to briefly consider the more general case of an overlapping-generations model, in which aggregate labor and dividend income follow an ergodic path. To generate a large equity premium in models of this kind it is necessary to assume not only undiversifiable risk in labor income, but also borrowing constraints similar to those examined by Constantinides et al. (1998). In this context, the risk reduction associated with government holdings of equity would be similar to that derived above, but the optimal policy would not, in general, require budget balance in every period. Rather, the government would pursue a tax-smoothing policy subject to constraints on net debt. This observation reinforces the point that the risk-reduction benefits from diversification are independent of the particular tax policy used to achieve long-run budget balance. It is also important to note that a diversification policy is not vulnerable to Croushore's second criticism of BMZ: that, in a multiperiod model, it is not obvious how to identify the "current" period in which a deficit should be used to generate "future" risk reductions.

APPENDIX

PROOF OF PROPOSITION 1:

It is convenient to express C_i in the following way:

$$C_i = D + \bar{Y} - s + k(D, g^e, p/q)\varepsilon_i$$

where

$$k(D, g_e, p/q) \equiv \frac{\bar{Y} + (D - p/q)g^e - s}{\bar{Y}}$$

$$\varepsilon_i \equiv Y_i - \bar{Y}.$$

Hence,

$$\mathrm{E}[u_2(C_i)]$$

$$= \mathrm{E}_D[\mathrm{E}_{\varepsilon_i}[u_2(D + \bar{Y} - s + k(D, g^e, p/q)\varepsilon_i)]]$$

$$= \mathrm{E}_D[u_2(c_{\varepsilon_i}(D + \bar{Y} - s + k(D, g^e, p/q)\varepsilon_i))]$$

where

$$c_{\varepsilon_i}(D + \bar{Y} - s + k(D, g^e, p/q)\varepsilon_i)$$

$$= u_2^{-1}(\mathrm{E}_{\varepsilon_i}[u_2(D + \bar{Y} - s + k(D, g^e, p/q)\varepsilon_i)])$$

is the *certainty equivalent* wealth, *conditional* on the value of D. Differentiating with respect to g^e yields

$$\frac{\partial}{\partial g^e}\mathrm{E}[u_2(C_i)]$$

$$= \mathrm{E}_D\left[u_2'(C_{\varepsilon_i}(D + \bar{Y} - s + k(D, g^e, p/q)\varepsilon_i))\right.$$

$$\times \frac{\partial}{\partial k}c_{\varepsilon_i}(D + \bar{Y} - s + k\varepsilon_i)_{|k = k(D, g^e, p/q)}$$

$$\left.\times \frac{\partial}{\partial g^e}k(D, g^e, p/q)\right].$$

Decreasing absolute risk aversion means that

$$\frac{\partial}{\partial k}c_{\varepsilon_i}(D + \bar{Y} - s + k\varepsilon_i)$$

is an increasing but negatively valued function of D. Also, notice that

$$\frac{\partial}{\partial g^e}k(D, g^e, p/q) = \frac{D - p/q}{\bar{Y}}$$

is increasing in D and

$$\mathrm{E}_D\left[u_2'(C_{\varepsilon_i}(D + \bar{Y} - s + k(D, g^e, p/q)\varepsilon_i)) \times \frac{\partial}{\partial g^e} k(D, g^e, p/q)\right]$$

$$= \mathrm{E}_D\left[u_2'(c_{\varepsilon_i}(D + \bar{Y} - s + k(D, g^e, p/q)\varepsilon_i)) \times \frac{D - \mathrm{E}[Du_2'(C_i)]/\mathrm{E}[u_2'(C_i)]}{\bar{Y}}\right]$$

$$\leq \frac{\mathrm{E}[Du_2'(C_i)] - \mathrm{E}[u_2'(C_i)]\mathrm{E}[Du_2'(C_i)]/\mathrm{E}[u_2'(C_i)]}{\bar{Y}}$$

$$= 0.$$

since decreasing absolute risk aversion implies $u''' \geq 0$.

Hence we have

$$\frac{\partial}{\partial g^e} \mathrm{E}[u_2(C_i)]_{|g^e=0}$$

$$> \mathrm{E}_D\left[\frac{\partial}{\partial k} c_{\varepsilon_i}(D + \bar{Y} - s + k\varepsilon_i)_{|k=k(D,g^e=0,p/q)}\right]$$

$$\times \mathrm{E}_D\left[u_2'(c_{\varepsilon_i}(D + \bar{Y} - s + k(D, g^e, p/q)\varepsilon_i)) \times \frac{\partial}{\partial g^e} k(D, g^e, p/q)\right] = 0.$$

It remains to show that risk vulnerability implies that an increase in g^e is accompanied by an increase in p/q. To show this, first note that from (9) and (10) we have

$$\text{(A1)} \qquad \frac{p}{q} = \frac{\mathrm{E}[Du_2'(C_i)]}{\mathrm{E}[u_2'(C_i)]}.$$

Hence it is sufficient to show that the partial derivative of the right-hand side of (A1) with respect to g_e is positive. Define

$$\mathrm{sgn}(x) = \begin{cases} + & \text{if } x > 0 \\ - & \text{if } x < 0 \end{cases}$$

$$\mathrm{sgn}\left(\frac{\partial}{\partial g_e}\left(\frac{\mathrm{E}[Du_2'(C_i)]}{\mathrm{E}[u_2'(C_i)]}\right)\right)$$

$$= \mathrm{sgn}\left(\mathrm{E}[u_2'(C_i)]\frac{\partial}{\partial g_e}(\mathrm{E}[Du_2'(C_i)]) - \mathrm{E}[Du_2'(C_i)]\frac{\partial}{\partial g_e}(\mathrm{E}[u_2'(C_i)])\right)$$

$$= \mathrm{sgn}\left(\mathrm{E}\left[\mathrm{E}[u_2'(C_i)]D - \mathrm{E}[Du_2'(C_i)])\frac{\partial}{\partial g_e}u_2'(C_i)\right]\right)$$

$$= \mathrm{sgn}\left(\mathrm{E}\left[\mathrm{E}[u_2'(C_i)]D - \mathrm{E}[Du_2'(C_i)])u_2''(C_i)\frac{(D - p/q)}{\bar{Y}}\varepsilon_i\right]\right)$$

$$= \mathrm{sgn}(\mathrm{E}_D[(D - p/q)^2(\mathrm{E}_{\varepsilon_i}[u_2''(C_i)\varepsilon_i])]).$$

But since $u''' > 0$ it follows that $\mathrm{E}_{\varepsilon_i}[u_2''(C_i)\varepsilon_i] > 0$ for all D and so

$$\mathrm{E}_D[(D - p/q)^2(\mathrm{E}_{\varepsilon_i}[u_2''(C_i)\varepsilon_i])] > 0$$

as required.

PROOF OF COROLLARY 2:

Notice that

$$\frac{d}{dg^e}(1 - T)_{|g^e=0} = \frac{(D - p/q)}{\bar{Y}}.$$

Property (15) implies that $\bar{Y}$ is nondecreasing in D and for any $d > d'$ we have $(Y_i - \bar{Y})|(D = d')$ is a mean-preserving spread of $(Y_i - \bar{Y})|(D = d)$. Hence, relative to the case of independence considered in Proposition (1), the reductions in variation of C_i for *low* realizations of D (that is, for $D < p/q$) are *larger*, and the increases in variation of C_i for *high* realizations of D (that is, for $D > p/q$) are *smaller*. Hence for preferences that exhibit decreasing absolute risk aversion the result holds as required.

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