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On relative performance contracts and fund manager's incentives

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Abstract

For pension schemes, mutual funds, banks and other financial intermediaries, large portfolio decisions are increasingly delegated to fund managers. Recently, there has been growing concern that these managers seem to adopt extremely similar investment strategies. One possible explanation for this phenomenon may be found in reward schemes based on relative performance. We show how relative performance reward schemes may arise as optimal contracts. Our focus is the fund owners–fund manager relationship in which the manager, before making a portfolio decision on behalf of the owners, may acquire, at some cost, information that is not available to the owners. Payment schemes based on relative performance afford the owners tighter control of their manager's activities. However, if two managers of different funds both accept contracts that depend on their relative as well as absolute performances, then there may exist equilibria in the managers' subgame that result in undesirable outcomes for the owners. © 1998 Elsevier Science B.V. All rights reserved.

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1. Introduction

'Managed funds' investments are an important form of asset ownership in many countries. For example, in the United States at the end of 1990, pension funds held approximately \$750 billion in equities, accounting for more than one-fifth of stock market capitalization. At the same time, it is estimated that another \$250 billion was invested in equity-oriented mutual funds, while altogether \$6.1 trillion of US financial assets were held by institutions who employ professional funds managers.[1] In Canada, total investment in mutual funds is over \$60 billion and in Australia pension fund assets total over \$100 billion.[2]

While 'managed funds' investments have been growing in importance, the performance of these funds has been strongly criticized. Lakonishok et al. (1992), for example, examine the performance of equity investments in US-defined benefit pension funds and conclude that the managers 'seem to subtract rather than add value relative to the performance of the *S&P* 500 *Index*' (p. 378; see also Malkiel, 1995; Goggin et al., 1993). Empirical evidence also suggests that fund managers actively 'window-dress' their portfolios (Lakonishok et al., 1991) and that they may 'herd' on similar assets (e.g. Grinblatt et al., 1995; Falkenstein, 1996). In a recent symposium on public policy issues in finance (Leland et al., 1997), Robert Glauber argues that one of the systemic causes of the 1987 stock market crash was the generally insufficient cash holdings of mutual funds (p. 1187) which he attributes to 'competitive pressure for performance and fear of ridicule' (p. 1188).

Fund managers are both implicitly and explicitly rewarded on the basis of relative performance. Poor relative performance also increases the probability that a fund manager will be replaced (Khorana, 1996). Some analysts claim that poor managed funds performance can be linked to these incentive contracts. For example, Davanzo and Nesbitt (1987) and Kritzman (1987) point out the opportunities for funds managers to 'game' incentive contracts. These claims stand in contrast to the conclusions of recent research on relative performance evaluation within principal/agent relationships.[3] In general, this research has shown that comparative performance evaluation can only *improve* agent performance. A principal can use relative performance measures to lower the cost of inducing desired behavior by an agent and/or to improve the action implemented. At worst, the principal can merely revert to the uncontingent contract.

A potential criticism of these theoretical results is that the models do not capture either the complexity of the fund manager's task or the importance of

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[1] See Goggin et al. (1993), Lakonishok et al. (1991, 1992).

[2] See Berkowitz and Kotowitz (1993) and Klumpes (1991), respectively.

[3] Holmström (1982) provides a useful introduction to this research. For a more general survey, see Hart and Holmström (1987).

uncoordinated decision making by multiple principals. Investors in a managed fund face a two-stage incentive problem. First, they require the fund manager to investigate relevant investment options; and secondly, they want the manager to choose a portfolio in accordance with their particular wishes. If fund owners wish to use relative performance evaluation to motivate their manager in this two-stage task then they must allow for the fact that other fund owners may also be pursuing similar strategies.

Some of the existing theoretical literature focuses on certain aspects of this problem. For example, Cohen and Starks (1988) consider the problem of inducing correct managerial portfolio choice (see also Starks, 1987; Grinblatt and Titman, 1989; Golec, 1992). These papers do not address explicitly the issue of multiple funds but rather focus on comparing single managers to the 'market' outcome. Also, the papers do not consider optimal contracts. Grinold and Rudd (1987) briefly consider the problem of a single manager with multiple clients while Brown et al. (1996) consider portfolio risk manipulation by fund managers who are likely to be relative performance 'losers'. Grinold and Rudd do not present a formal model. Brown et al. concentrate on empirical testing of portfolio manipulation rather than developing a formal fund manager model.

Our aim in this paper is both to formalize and to analyze the 'fund manager problem'. We first consider the two-stage incentive problem and characterize the optimal relative performance contracts that fund owners can use to motivate their managers. The situation where both principals set relative performance contracts, however, may not be robust. By focusing on a specific information structure, we show that if managers simultaneously face relative performance contracts, then the outcome where both agents act in accord with their investors' wishes may be only one of a number of equilibrium managerial choices. Further, this equilibrium may not survive the iterated deletion of weakly dominated strategies. Put simply, the naive use of relative performance evaluation is likely to lead to unintended managerial activity. These equilibrium actions can involve either too little or too much risk taking by fund managers. Outcomes may also involve managerial 'herding'. In contrast to the existing literature which focuses on herding due to either signal jamming between different types of managers (Scharfstein and Stein, 1990), inefficient information transmission (Banerjee, 1992; Bikhchandani et al., 1992; Welch, 1992) or free-riding in information gathering (King, 1995), herding may occur when the managers choose an equilibrium which, from the investors' perspective, is undesirable.

Of course, if fund owners realize that the simultaneous setting of relative performance contracts will not result in desired managerial behavior, then they will not naively write these contracts. In the final part of the paper, we consider the game played between the owners of two different funds. Each group of owners prefers to use relative performance evaluation if the other group omits such evaluation from their managerial contracts and vice versa. Pure strategy equilibria will involve asymmetric contracts between funds. More importantly, there will also be a mixed strategy equilibrium which potentially involves suboptimal managerial performance. This equilibrium thus endogenises the concerns expressed in the literature about fund manager performance.

2. The environment

In this paper, funds are viewed as agencies which collect contributions of their members to invest them on their behalf.[4] Throughout we shall assume there are two independent funds, 1 and 2. These funds need managers who monitor financial markets and carry through the necessary investment activities. The fact that a manager performs the monitoring action gives her information which is unavailable to the fund members. Depending on the reward structure of the manager, she may find it optimal to manipulate this private information to her advantage. A reasonable reward structure for the manager must therefore take account of these information constraints. In this context, the question arises to what extent relative performance of a fund becomes a potential control mechanism for the fund owners.

To abstract from the problem of divergent incentives of the fund owners, it is assumed that owners of a particular fund have identical preferences. This abstraction allows us to treat the fund owners as a single representative agent, the *principal*. To further simplify exposition, it is assumed that fund owners are risk-neutral. As will be demonstrated below, the incentive problem remains unchanged if a risk-averse fund owner is considered.

Two assets are available as investment opportunities. There is a bond which has a state-independent return I and an asset with a state-dependent return $r(\cdot)$. Throughout this paper it will be assumed that the fund wishes to invest a fixed amount W.[5] If we denote by δ the fraction of these funds invested in the risky asset, then the return from a portfolio $\delta \in [0, 1]$ for fund i ($i = 1, 2$) in state ω may be expressed as

$$R_i(\omega, \delta) = [I + (r(\omega) - I)\delta]W. \tag{1}$$

Each group of fund owners want its manager to choose a portfolio δ that maximizes the expected revenue from this portfolio minus the expected payment to its manager. The fund owners are assumed to observe costlessly only the actual returns of the two funds. Therefore, payments to the manager can be based on the observed returns of the two funds only. In particular, payments cannot be made contingent on a manager's information activities and her

[4] In other words, we are dealing with accumulation funds, where the investor's return is variable.
[5] Attraction of new contributions to the fund is not the issue of this paper.

portfolio choice directly. This restriction on the information of fund owners tries to capture the fact that, though not strictly impossible, it may be extremely costly for a large number of fund owners to obtain detailed information about their own manager's activity let alone the activities of a manager of another fund.

Each fund manager is assumed to be a risk-averse expected-utility maximizer. In a first stage, each manager has to collect information and, in a second stage, in the light of the information obtained she has to choose the portfolio. Gathering of information is modelled as a binary choice

- to undertake information activities at a fixed cost e in terms of the manager's expected utility and receive a signal, or
- not to seek information.

Information about the investment opportunities may include past performance records of these investments and data about the general economic environment that has an impact on success or failure of an investment project. Such information is called a *signal*. Without gathering such relevant information a fund manager has to base her investment decision on a prior belief about the possible returns on her investment. After obtaining the relevant information, i.e. after receiving a signal, the fund manager can update her beliefs. A signal provides information because it is jointly distributed with the state of the world.

Throughout this paper, we will assume that there are just two relevant events for the risky investment opportunity, which correspond to a 'good' outcome (G) and a 'bad' result (B). Similarly, from the information-gathering process of each manager only two possible signal realizations may arise, a 'high' realization indicating favorable news about the risky asset and a 'low' realization suggesting unfavorable information. We shall denote H (respectively, L) as the high (respectively, low) signal realization for the manager of the first fund. Finally, h and l shall denote, respectively, the high and low signal realizations for the manager of the second fund.

A characteristic feature of the owner–manager relationship is the fact that the information-gathering activity of the fund manager is unobservable and thus represents a moral hazard problem, while carrying out this required task creates private information of the manager about the quality of the risky asset and, therefore, a hidden information problem when allocating the funds of the owners.

Let us denote a contract offered by the owners of fund i, as a contingent payment schedule $x_i(\cdot, \cdot)$ where the first argument is the return of the portfolio managed by the manager of fund i and the second argument is the *difference* between the returns of the manager's and the other fund's portfolios. In general, such a payment schedule may be viewed as a relative performance contract since the payment to the manager depends not only on the absolute performance of the portfolio managed by her but also on the performance of her portfolio relative to the performance of the other fund's portfolio. A non-relative (i.e. absolute) performance contract is simply a payment schedule which does not vary in the second argument.

The sequence of actions by the two principals and the two managers may be summarized as follows:

1. The two groups of fund owners simultaneously propose contracts, $x_i(\cdot, \cdot)$ to their respective managers.
2. The managers simultaneously decide whether to accept or reject their respective offers. If a manager rejects the contract offered to her, then she achieves an expected utility level v from her outside opportunities.[6]
3. If a manager accepts the contract offered to her, she chooses first whether to gather information or whether to act upon her prior information.
4. Each manager makes a portfolio decision given her information.
5. Uncertainty is revealed, returns are realized, and the contracted payments to the managers are made.

A few assumptions on the return distribution of the risky asset simplify the exposition and rule out trivial cases where no contracts would be feasible.

Assumption 1. (i) The returns of the assets are $I = 0$ for the riskless bond and

$$r(\omega) = \begin{cases} 1 & \text{if } \omega \in G \\ -1 & \text{if } \omega \in B \end{cases} \quad \text{for the risky asset.}$$

(ii) The joint probability distribution over returns and each signal satisfy

$$\Pr(G \cap H) > \Pr(B \cap H), \qquad \Pr(B \cap L) > \Pr(G \cap L),$$

$$\Pr(G \cap h) > \Pr(B \cap h), \qquad \Pr(B \cap l) > \Pr(G \cap l).$$

(iii) The joint probability distribution over the two signals satisfy

$$\Pr(H \cap h) > \Pr(H \cap l), \qquad \Pr(L \cap l) > \Pr(L \cap h),$$

$$\Pr(h \cap H) > \Pr(h \cap L), \qquad \Pr(l \cap L) > \Pr(l \cap H).$$

The numerical specification of the returns in Assumption 1(i) simplifies exposition and does not reduce the generality of the results. In fact, one may think of these return rates as deviations from the certain return rate. Even the degree of deviation is immaterial for the form of the incentive contracts as will be

[6] Strictly speaking, the offer of each group of fund owners to its manager should include an alternative absolute performance contract in the event that its manager accepts the contract while the (prospective) manager of the other rejects the offer made to her.

proved formally below. The second part of the assumption simply formalizes the notion of each manager's signal being informative. The third part of the assumption entails that each manager's signal is informative about the other manager's signal. As we shall see in the next section, given the belief that the other manager will acquire and use the information provided by her signal, this provides the incentive for fund owners to make the contract they offer their manager contingent on the relative performance of her portfolio.

With this numerical specification of returns, the return from a portfolio δ for fund $i \in \{1, 2\}$ simplifies to

$$R_i(\omega, \delta) = \begin{cases} \delta W & \text{if } \omega \in G, \\ -\delta W & \text{if } \omega \in B. \end{cases} \tag{2}$$

It is obvious that a risk-neutral fund owner would like the manager to invest all capital into the risky asset ($\delta = 1$) if the probability of a good event is higher than the probability of the bad event. If this is not the case, the owners would want no investment in the risky asset at all ($\delta = 0$).

At this stage the value of the signal for the owners becomes clear. To rule out the uninteresting case where the fund owners have no interest in the available information, it is assumed that the joint probability distribution over returns and each signal is such that, under full information each group of fund owners would want to get the signal and would want to invest into the risky asset if and only if the signal's realization is high. This condition is formalized in the following assumption. For any event E in $\{G, B\}$ and any signal realization y in $\{H, L\} \cup \{h, l\}$, denote by $\Pr(E|y) := \Pr(E \cap y)/\Pr(y)$ the updated probability of the event E given the signal realization y.

Assumption 2. The following conditions on the joint probability distribution are assumed to hold:

$$\Pr(H)[\Pr(G|H) - \Pr(B|H)]W - e > \max[0, \Pr(G) - \Pr(B)]W,$$

$$\Pr(h)[\Pr(G|h) - \Pr(B|h)]W - e > \max[0, \Pr(G) - \Pr(B)]W.$$

3. Characterizing the relative performance contracts

Denote by $(\alpha_i, \beta_i) \in [0, 1] \times [0, 1]$, the investment strategy by manager i that involves investing a portion α_i (respectively, β_i) of the available funds into the risky asset if the signal's realization received by manager i is high (respectively, low). Given the risk neutrality of the fund owners and the assumption on the information structure (Assumption 2), it is clear that both groups of fund owners would ideally like to write contracts that induce their respective managers to gather information, to invest all funds into the risky asset if the signal's realization is high, and to invest exclusively in the riskless asset if the low signal realization comes up. That is, if it is not too expensive, fund owners wish their manager to implement the investment strategy (1, 0). The owners are, of course, interested in implementing this behavior at the lowest possible expected cost. Since there is another fund whose owners provide an incentive contract for their manager to invest according to the strategy (1, 0), owners of fund i may condition the payments to their manager on both the absolute return of their fund, $R_i(\omega, \delta_i)$ and the difference in the performances of the two funds, $R_i(\omega, \delta_i) - R_j(\omega, \delta_j)$, where δ_i (respectively, δ_j) is the portfolio decision made by manager i (respectively, j). To simplify notation, $x_i(\delta_i, \delta_i - \delta_j)$ (respectively, $x_i(-\delta_i, -\delta_i + \delta_j)$) will be written for the payout to fund manager i, if $(R_i(\omega, \delta_i), R_j(\omega, \delta_j))$ is observed where $\omega \in G$ (respectively, $\omega \in B$).

Given a relative performance contract x_1, a portfolio choice δ, and given that the manager of fund 2 is following the investment strategy (α_2, β_2), the conditional (on signal H and L) expected utility of the manager of fund 1 will be denoted by

$$\begin{aligned} V_1(\delta, x_1|H, (\alpha_2, \beta_2)) := {} & \Pr(B\cap h|H)\cdot u(x_1(-\delta, -\delta + \alpha_2)) \\ & + \Pr(B\cap l|H)\cdot u(x_1(-\delta, -\delta + \beta_2)) \\ & + \Pr(G\cap h|H)\cdot u(x_1(\delta, \delta - \alpha_2)) \\ & + \Pr(G\cap l|H)\cdot u(x_1(\delta, \delta - \beta_2)), \\ V_1(\delta, x_1|L, (\alpha_2, \beta_2)) := {} & \Pr(B\cap h|L)\cdot u(x_1(-\delta, -\delta + \alpha_2)) \\ & + \Pr(B\cap l|L)\cdot u(x_1(-\delta, -\delta + \beta_2)) \\ & + \Pr(G\cap h|L)\cdot u(x_1(\delta, \delta - \alpha_2)) \\ & + \Pr(G\cap l|L)\cdot u(x_1(\delta, \delta - \beta_2)), \end{aligned}$$

and the unconditional expected utility of the manager of fund 1 will be denoted by

$$\begin{aligned} V_1(\delta, x_1|(\alpha_2, \beta_2)) := {} & \Pr(H)\cdot V_1(\delta, x_1|H, (\alpha_2, \beta_2)) \\ & + \Pr(L)\cdot V_1(\delta, x_1|L, (\alpha_2, \beta_2)). \end{aligned}$$

For the manager of fund 2, the expected utilities $V_2(\delta, x_2|H, (\alpha_1, \beta_1))$, $V_2(\delta, x_2|L, (\alpha_1, \beta_1))$, and $V_2(\delta, x_2|(\alpha_1, \beta_1))$ are defined in an analogous fashion.

If the managers of both funds are following the investment strategy (1, 0), each fund can only get one of the three returns $\{-W, 0, W\}$. Since both managers are not supposed to invest in the risky asset if they observe the low signal realization, the returns for the events $G\cap L\cap l$ and $B\cap L\cap l$ will be the same. Thus, there can be 7 $(2^3 - 1)$ observationally distinct events on which the fund owners can condition their payoffs to the manager. Notice also that the owners of either fund can prevent their manager from choosing a portfolio $\delta \in (0, 1)$ as such a choice would be detected from the results of the funds and could be 'punished' by imposing a payoff that is worse than the worst payoff from contract-consistent behavior. It is therefore assumed that neither manager will ever choose a portfolio other than $\delta = 1$ or $\delta = 0$.

The following problem describes the choice situation of the owners of fund 1, designing a relative performance contract $x_1(\cdot, \cdot)$ that minimizes the expected cost of inducing their manager to follow the investment strategy (1, 0), given that the owners of fund 1 and their manager believe that the manager of fund 2 is following the investment strategy (1, 0). Let s_1 (respectively, t_1) denote the investment strategy (1, 0) for the manager of fund 1 (respectively, 2).

$$\min_{\langle x_1(\cdot,\ \cdot)\rangle} \quad \Pr(H)[\Pr(B \cap h|H) \cdot x_1(-1, 0) + \Pr(B \cap l|H) \cdot x_1(-1, -1)$$

$$+ \Pr(G \cap h|H) \cdot x_1(1, 0) + \Pr(G \cap l|H) \cdot x_1(1, 1)]$$

$$+ \Pr(L)[\Pr(B \cap h|L) \cdot x_1(0, 1) + \Pr(l|L) \cdot x_1(0, 0)$$

$$+ \Pr(G \cap h|L) \cdot x_1(0, -1)]$$

s.t.

$$V_1(1, x_1|H, t_1) \geq V_1(0, x_1|H, t_1), \qquad \text{(IC1)}$$

$$V_1(0, x_1|L, t_1) \geq V_1(1, x_1|L, t_1), \qquad \text{(IC2)}$$

$$\gamma_1\!: \ \Pr(H)V_1(1, x_1|H, t_1) + \Pr(L)V_1(0, x_1|L, t_1) - e \geq V_1(0, x_1|t_1), \quad \text{(WC1)}$$

$$\gamma_2\!: \ \Pr(H)V_1(1, x_1|H, t_1) + \Pr(L)V_1(0, x_1|L, t_1) - e \geq V_1(1, x_1|t_1), \quad \text{(WC2)}$$

$$\lambda\!: \ \Pr(H)V_1(1, x_1|H, t_1) + \Pr(L)V_1(0, x_1|L, t_1) - e \geq v. \qquad \text{(IR)}$$

Note that the Lagrangian multipliers γ_1, γ_2 and λ have been associated with the constraints (WC1), (WC2) and (IR), respectively.

The first two *incentive compatibility constraints* (IC1), (IC2) guarantee that after the manager has undertaken her information activity and received her signal the contract provides no incentive for her to deviate from the desired investment strategy s_1. The *work constraints* ensure a sufficient incentive for the manager to undertake her information activity rather than simply invest unconditionally either in the safe asset (WC1) or in the risky asset (WC2). Finally, the *individual rationality* (IR) (or participation constraint) secures the manager's acceptance of the contract (given her belief that the manager of fund 2 is following the investment strategy t_1).

The following lemma shows that the incentive constraints will be satisfied automatically for any solution to the problem subject to the work constraints.

Lemma 3. The work constraints (WC1) and (WC2) imply the incentive constraints (IC1) and (IC2).

Proof. It is easy to check the inequalities following the (WC1) constraint:

$$\begin{aligned}\Pr(H)V_1(1, x_1|H, t_1) + \Pr(L)V_1(0, x_1|L, t_1) &\geq V_1(0, x_1|t_1) + e \\ &\geq V_1(0, x_1|t_1) \\ &= \Pr(H)V_1(0, x_1|H, t_1) \\ &+ \Pr(L)V_1(0, x_1|L, t_1).\end{aligned}$$

This implies the IC1 constraint $V_1(1, x_1|H, t_1) \geq V_1(0, x_1|H, t_1)$. Similarly, one can prove that WC2 implies IC2. □

The following properties of the expected-cost minimizing contract are proved in the appendix.

Lemma 4. For any solution to this problem, λ, γ_1 and γ_2 are all strictly positive.

From Lemma 4 one concludes that the (WC1), (WC2) and the (IR) constraints are binding. Given these results, it is straightforward to derive the following first-order conditions for an interior solution of the optimal contract problem:

$$\frac{1}{u'[x_1^*(-1, 0)]} = \lambda + \gamma_1 - \gamma_2 \frac{\Pr(L|B\cap h)}{\Pr(H|B\cap h)}, \tag{3}$$

$$\frac{1}{u'[x_1^*(-1, -1)]} = \lambda + \gamma_1 - \gamma_2 \frac{\Pr(L|B\cap l)}{\Pr(H|B\cap l)}, \tag{4}$$

$$\frac{1}{u'[x_1^*(1, 0)]} = \lambda + \gamma_1 - \gamma_2 \frac{\Pr(L|G\cap h)}{\Pr(H|G\cap h)}, \tag{5}$$

$$\frac{1}{u'[x_1^*(1, 1)]} = \lambda + \gamma_1 - \gamma_2 \frac{\Pr(L|G\cap l)}{\Pr(H|G\cap l)}, \tag{6}$$

$$\frac{1}{u'[x_1^*(0, 1)]} = \lambda - \gamma_1 \frac{\Pr(H|B\cap h)}{\Pr(L|B\cap h)} + \gamma_2, \tag{7}$$

$$\frac{1}{u'[x_1^*(0, -1)]} = \lambda - \gamma_1 \frac{\Pr(H|G\cap h)}{\Pr(L|G\cap h)} + \gamma_2, \tag{8}$$

$$\frac{1}{u'[x_1^*(0, 0)]} = \lambda - \gamma_1 \frac{\Pr(H|l)}{\Pr(L|l)} + \gamma_2. \tag{9}$$

Eqs. (3)–(9) fully characterize the optimal contract when the fund owners use the information of a second fund's performance.[7] An analogous problem and set of first-order conditions characterizes the optimal relative performance contract $x_2^*(\cdot,\cdot)$ given the belief of the owners and manager of fund 2 that the manager of fund 1 is following the investment strategy s_1.

Remark. For any given strategy by the manager of fund 2 of the form $(\alpha_2, 0)$, α_2 in $(0, 1)$, the owners of fund 1 can utilize the basic structure of this solution to construct a payment scheme to implement any strategy of the form $(\alpha_1, 0)$ with α_1 in $(0, 1)$ for the *same* expected cost.

Corollary 5. Fix the beliefs of the fund owners and manager of fund 1 that the manager of fund 2 is following the investment strategy $(\alpha_2, 0)$, *with* α_2 in $(0, 1]$. *The optimal contract for the implementation of an investment strategy* $(\alpha_1, 0)$ *for some* α_1 in $(0, 1)$, *denoted* $x_1^{\alpha_1}(\cdot,\cdot)$, satisfies

$$x_1^{\alpha_1}(-\alpha_1, -\alpha_1+\alpha_2) = x_1^*(-1, 0), \qquad x_1^{\alpha_1}(-\alpha_1, -\alpha_1) = x_1^*(-1, -1),$$

$$x_1^{\alpha_1}(\alpha_1, \alpha_1-\alpha_2) = x_1^*(1, 0), \qquad x_1^{\alpha_1}(\alpha_1, \alpha_1) = x_1^*(1, 1),$$

$$x_1^{\alpha_1}(0, -\alpha_2) = x_1^*(0, -1), \quad x_1^{\alpha_1}(0, \alpha_2) = x_1^*(0, 1), \quad x_1^{\alpha_1}(0, 0) = x_1^*(0, 0).$$

The fact that the expected-cost minimizing contract for inducing the manager to invest, conditional on receiving the high signal realization, is invariant to the proportion of funds that the principal wishes to have invested in the risky asset *and* is invariant to the proportion of funds that the other manager invests in the risky asset conditional on receiving a high signal realization, allows us to solve the risk-averse fund owners' optimal contract recursively. One first computes the minimum expected-cost payment scheme for the risk-neutral fund owners and then determines the risk-averse fund owners' optimal investment strategy that they wish the manager to follow. Denoting $v_1(\cdot)$ the von Neumann–Morgenstern utility function of the representative owner of fund 1, and $(\alpha_2, 0)$ the investment strategy employed by the manager of fund 2, $\alpha_1 \in [0, 1]$ is chosen to maximize

$$\begin{aligned} V_1(\alpha_1) = \Pr(H)\cdot[&\Pr(B\cap h|H)\cdot v_1(-\alpha_1\cdot W - x_1(-1,0)) \\ &+ \Pr(G\cap h|H)\cdot v_1(\alpha_1\cdot W - x_1(1,0)) \\ &+ \Pr(B\cap l|H)\cdot v_1(-\alpha_1\cdot W - x_1(-1,-1)) \end{aligned}$$

[7] Strictly speaking, we need to add that the manager is 'punished' (i.e. receives some minimum payment) if any combination of absolute performance and relative performance apart from those explicitly referred to in Eqs. (3)–(9) is observed.

$$+ \Pr(G \cap l|H) \cdot v_1(\alpha_1 \cdot W - x_1(1, 1))]$$

$$+ \Pr(L) \cdot [\Pr(B \cap h|L) \cdot v_1(0 - x_1(0, 1))$$

$$+ \Pr(l|L) \cdot v_1(0 - x_1(0, 0))$$

$$+ \Pr(G \cap h|L) \cdot v_1(0 - x_1(0, -1))].$$

Returning to our original case where the owners of fund 1 wish to implement the investment strategy (1, 0), we can interpret Eqs. (3)–(9) as saying that the fund owners want to differentiate the payments to the manager in response to the second fund's results whenever this information is valuable. Information about the second fund's performance is only useless if the relative likelihood of a signal for the first manager is not affected by the signal that the second fund manager receives, e.g., if $\Pr(L|B \cap h)/\Pr(H|B \cap h) = \Pr(L|B \cap l)/\Pr(H|B \cap l)$. In this case, one can conclude from Eqs. (3) and (4) that $x_1^*(-1, 0) = x_1^*(-1, 1)$ is optimal. Hence, the payment to the first fund manager for a bad result need not be differentiated according to the other fund's performance.

In general, however, observation of the returns of the other fund provides information about the signal which the first fund manager observes. For example, if $(R_1(\omega, \delta_1), R_2(\omega, \delta_2)) = (0, 1)$ were observed, the owners of fund 1 could infer that fund 2's manager observed the high signal realization h and that the good event came about. Without observing the second fund's result, this information would be unavailable for the fund owners since their own manager decided to invest in the safe asset ($\delta_1 = 0$). This is valuable information for the fund owners, because, given the belief that the other fund's manager has worked and is following the investment strategy t_1, it is less likely that their manager would observe a low signal realization L if event G and signal realization h occurred. Thus, writing a contract that 'punishes' their own manager for such an outcome enables the principal to induce the desired behavior at a lower cost. Whether a low signal realization is more or less likely than a high signal realization, given the information from the other fund, depends on the ratio $\Pr(L|\cdot)/\Pr(H|\cdot)$, which becomes therefore a crucial parameter for the payment structure.

For an interpretation of the optimal contract structure, recall that γ_1 is the multiplier for the constraint that the manager should not want to shirk by deviating to the investment strategy (0, 0), while γ_2 is the multiplier for the constraint which prevents the manager from shirking by choosing the investment strategy (1, 1). Payments to the manager consist of a base income to compensate for the effort and the opportunity costs plus an incentive adjustment to induce the fund manager to acquire information rather than choosing an investment plan without this information. Information about the second fund's performance is important for structuring the adjustments to provide the required incentives in the least possible expected-cost way.

4. Mutual relative performance contracts and the managers' subgame

The results of the previous section suggest that it is in general worthwhile for fund owners to use relative performance indicators for the design of a remuneration contract for their manager. The additional information obtained from the observed returns of the other fund is valuable for the fund owners in the design of their incentive contract. This result rests, however, on the presumption that the other fund manager obeys an incentive contract which makes it optimal for her to act in the interest of her fund owners.

If both managers have accepted from their respective fund owners relative performance contracts of the form characterized in the previous section, then it is straightforward to show (as we do below) that in the ensuing subgame, it is an equilibrium for both managers to engage in information gathering and utilize that information in the way that their relative performance contracts had intended. But *simultaneous* deviations by the two managers from this intended behavior changes the information contained in the observed results. The issue that needs to be addressed is whether there are other strategy combinations for the two managers that form a Nash equilibrium in this subgame. The robustness of the equilibrium corresponding to the behavior of the two managers that the two relative performance contracts is intended to implement may be called into question if the payoff for each of the fund managers in an alternative equilibrium of their subgame exceeds that of the intended equilibrium.

To make the above points clearer, let us assume that both groups of fund owners have written a relative performance contract of the form derived in Section 3. If the managers both accept their respective contracts and if we consider only investment policies $\delta \in \{0, 1\}$, then each manager has four investment strategies: $\{s_1, s_2, s_3, s_4\}$ for manager 1 and $\{t_1, t_2, t_3, t_4\}$ for manager 2. These strategies are given in Table 1.

The strategies $\{s_3, s_4\}$ and $\{t_3, t_4\}$ do not require that the managers actually gather information, saving them the effort costs.

Given these strategies and the payment structure of the optimal contract, it is straightforward, though somewhat tedious, to derive the expected payoff to the managers from any strategy combination. Denote by π_{ij} and p_{ij} the payoffs of managers 1 and 2, respectively, if the strategy combination (s_i, t_j) is chosen. The payoff matrix describes the resulting game as shown in Table 2.

Table 1
Strategies

	s_1	s_2	s_3	s_4		t_1	t_2	t_3	t_4
$\delta(H)$	1	0	0	1	$\delta(h)$	1	0	0	1
$\delta(L)$	0	1	0	1	$\delta(l)$	0	1	0	1

Table 2
Payoff matrix

		Manager 2			
		t_1	t_2	t_3	t_4
Manager 1	s_1	π_{11}, p_{11}	π_{12}, p_{12}	π_{13}, p_{13}	π_{14}, p_{14}
	s_2	π_{21}, p_{21}	π_{22}, p_{22}	π_{23}, p_{23}	π_{24}, p_{24}
	s_3	π_{31}, p_{31}	π_{32}, p_{32}	π_{33}, p_{33}	π_{34}, p_{34}
	s_4	π_{41}, p_{41}	π_{42}, p_{42}	π_{43}, p_{43}	π_{44}, p_{44}

The incentive constraints of the two relative performance contracts guarantee that (s_1, t_1), the contractual intended behavior, forms a Nash equilibrium. To see this, notice that for the given strategy t_1, the relative performance contract $x_1^*(\cdot,\cdot)$ derived in Section 3 satisfies (according to Lemma 4)

- $\pi_{11} = v$ (as the (IR) constraint binds),
- $\pi_{11} = \pi_{31}$ and $\pi_{11} = \pi_{41}$ (as both the work constraints (WC1) and (WC2) bind),
- $\pi_{11} \geq \pi_{21}$ (by the incentive constraints (IC1) and (IC2), which by Lemma 3 are implied by the work constraints (WC1) and (WC2).

Hence, s_1 is a best response for manager 1 given that manager 2 chooses t_1. Similarly from the analogous derivation of $x_2^*(\cdot, \cdot)$, t_1 is a best response for manager 2 given that manager 1 chooses s_1. This shows that (s_1, t_1) is a Nash equilibrium in the managers' subgame that follows their simultaneous acceptance of their respective relative performance contracts.

Notice, however, that (s_1, t_1) is not a strict Nash equilibrium. As the relative performance contracts are designed to extract all the available surplus in the respective principal–agent relationships, by construction s_3 and s_4, as well as the intended behavior s_1, are best responses to t_1. Similarly, t_3 and t_4 are also best responses to s_1. Furthermore, note that the incentive constraints do not restrict the payoffs of strategy combinations in the submatrix below and to the right of (s_1, t_1). For example, as we demonstrate in the next section, it is quite possible to have equilibria like (s_3, t_3) for the managers subgame, where both managers do not seek information and choose the investment strategy $\beta = 0$. Such a strategy combination deprives the fund owners of all information and guarantees each manager i the certain payoff x_i*(0, 0) without incurring any effort costs.

It is difficult to give necessary and/or sufficient conditions for the existence of specific alternative equilibria, but we conjecture that typically there will be other equilibria where both managers choose not to seek information and thereby save on their effort costs and that as the behavior (s_1, t_1) intended by the relative performance contracts extracts all the surplus from the managers, the intended equilibrium (s_1, t_1) will not survive the refinement of iterated elimination of weakly dominated strategies.

While we do not prove this conjecture generally, in the next section we report that it holds for a specific parameterization of the prior probability distribution given a particular functional form for the managers' von Neumann–Morgenstern utility which allows us to calculate explicitly the relative performance contract derived in Section 3.

5. Symmetric risk preferences and information processing

As the two groups of fund owners in our environment have essentially identical features, an additional natural restriction to impose is to make the managers identical in risk attitudes and have the same ability to conduct information gathering activities. To be able to solve the system of equations in Section 3 that characterize the relative performance contract, it is convenient to assume both managers' utility function is $u(x) = \sqrt{2x}$. For this utility function we have $1/u'(x) = u(x)$, hence the first-order conditions in Eqs. (3)–(9) become linear in utilities and the Lagrangian multipliers. The equal ability of the two managers is embodied in the following restrictions on the joint probability space.

Symmetry of signals: The joint probability distribution over each manager's signal and the state space satisfies

$$\text{(1)}\ \Pr(G, H) = \Pr(G, h) \Rightarrow \Pr(G, H, l) = \Pr(G, L, h) \Rightarrow \Pr(G, l) = \Pr(G, L);$$

$$\text{(2)}\ \Pr(B, L) = \Pr(B, l) \Rightarrow \Pr(B, L, h) = \Pr(B, H, l) \Rightarrow \Pr(B, h) = \Pr(B, H).$$

This in turn implies $\Pr(H) = \Pr(h)$ $(\Pr(L) = \Pr(l))$.

The next assumption requires the prior belief of the likelihoods of the good and bad events to be equal and that the posterior beliefs after the receipt of the signal lie 'equally distant' either side of this prior.

Balanced signal: The unconditional probability that manager 1 receives the high signal realization is $\frac{1}{2}$ and the ex interim probability that the signal realization is informative of the true event is the same whether the signal realization is high or low, i.e.

$$1 > \Pr(G|H) = \Pr(B|L) > \tfrac{1}{2} > \Pr(B|H) = \Pr(G|L) > 0$$

and

$$\Pr(H) = \tfrac{1}{2}.$$

Table 3
Probability distribution

	G			B	
	h	l		h	l
H	$\theta\rho/2$	$(1-\rho)/4$	H	$(1-\theta)\rho/2$	$(1-\rho)/4$
L	$(1-\rho)/4$	$(1-\theta)\rho/2$	L	$(1-\rho)/4$	$\theta\rho/2$

Symmetry and balanced signal imply $\Pr(B, L, l) - \Pr(B, H, h) = \mathrm{P}(G, H, h) - \Pr(G, L, l)$.

It readily follows that the only distributions that satisfy both symmetry and balanced signal can be parameterized by two numbers $\rho, \theta \in (\frac{1}{2}, 1]$ and can be represented in the following probability Table 3.

We refer to ρ as the *correlation parameter* since it measures the correlation between the investment strategies of both managers if they both acquire the signal and employ the strategy (1, 0). That is, ρ is the probability that the managers do the same thing, or equivalently, given they are both following the investment strategy (1, 0), that their signals 'agree'. We refer to θ as the *accuracy parameter* as it measures, given that the managers are both following the investment strategy (1, 0) and that their signals agree, the probability that their investment strategy is correct.

Symmetry along with the other specifications in Section 2 entail that the optimal relative performance contracts are the same for both groups of fund owners, i.e. $x_1^*(\cdot, \cdot) = x_2^*(\cdot, \cdot)$.

The first-order conditions of the program defined in Section 3 that determines the optimal relative performance contract form a system of ten equations (Eqs. (3)–(9) plus the binding (IR), (WC1), and (WC2) constraints). This system is characterized by five parameters (ρ, θ, W, e, v), the correlation of the two signals, their accuracy, the amount each fund has available to invest, the effort cost for each manager to engage in information gathering and each manager's reservation utility. With the utility function $u(x) = \sqrt{2x}$ these equations are linear in the seven values of $u(x_1^*(\cdot, \cdot))$ and the system can be solved for the values of $u(x_1^*(\cdot, \cdot))$. Given the solution $u(x_1^*(\cdot, \cdot))$ to the set of ten equations above, one simply sets $x_1^*(\cdot, \cdot) := [u^*(\cdot, \cdot)]^2/2$. We have computed $u^*(\cdot, \cdot)$ (and $x_1^*(\cdot, \cdot)$) for a range of parameter values using a Mathematica program.[8]

[8] This Mathematica program is listed as Appendix B of a longer version of this paper which is available on the Internet under http://www.uni-sb.de/rewi/fb2/eichberger/english/eichberger/papers.htm.

Table 4
Payoff matrix

		Manager 2			
		t_1	t_2	t_3	t_4
Manager 1	s_1	10.00	9.71	9.94	9.77
	s_2	8.00	9.55	9.09	8.46
	s_3	10.00	10.86	12.93	7.94
	s_4	10.00	10.40	8.11	12.29

General properties of the solution $x_1^*(\cdot,\cdot)$ $(\equiv x_2^*(\cdot,\cdot))$ are hard to discern, but after considering a wide range of the parameter values and investigating the associated payoff matrices that the two managers face in the subgame after they have accepted the contract but before they have decided to expend the effort necessary to acquire the signal, we have identified two distinct cases. First, if we consider a relatively low θ, s_1 is weakly dominated by a half–half mixture of s_3 and s_4. Hence, the only three equilibria that survive iterated deletion of weakly dominated strategies (where domination by mixtures of other strategies is allowed) are (s_3, t_3), (s_4, t_4) and the associated mixed strategy. A typical example of case 1 arises for the parameter values $\rho = 0.6$, $\theta = 0.7$, $W = 1000$, $e = 1$ and $v = 10$. The resulting payoff matrix for manager 1 appears in Table 4. Manager 2's payoff matrix is obtained as the transpose of this matrix.

The second case arises for sufficiently high θ in which s_1 is no longer dominated by any combination of s_3 and s_4 but the only equilibria to survive iterated domination of weakly dominated strategies is (s_3, t_3). In this case, s_4 is weakly dominated by s_3 and s_2 is strictly dominated by s_1. By symmetry it obviously follows that t_4 and t_2 are also weakly dominated. But after the deletion of those four weakly dominated strategies, in the remaining 2×2 game, s_1 (respectively, t_1) is weakly dominated by s_3 (respectively, t_3). The parameter configuration $\rho = 0.6$, $\theta = 0.9$, $W = 1000$, $e = 1$ and $v = 10$ provides an example of case 2 with the payoff matrix of Table 5.

Games with multiple equilibria are notoriously difficult to analyze. Many refinements have been proposed in the game-theoretic literature.[9] Most of these refinements suggest that Nash equilibria in which some players use weakly dominated strategies are not robust. Experimental evidence is less unanimous on this point. Experiments by Camerer (1997) indicate however that at least two rounds of elimination of dominated strategies can be safely assumed.

[9] Van Damme (1987) provides an excellent treatment of these refinements.

Table 5
Payoff matrix

		Manager 2			
		t_1	t_2	t_3	t_4
Manager 1	s_1	10.00	10.38	10.44	9.94
	s_2	8.00	8.60	9.30	7.30
	s_3	10.00	11.25	11.52	9.72
	s_4	10.00	9.73	10.21	9.52

From an economic point of view, the multiple equilibria are a representation of the struggle between fund owners and fund managers for the informational surplus. It is in the nature of an optimal contract for the fund owner that s/he will not pay more to the manager than is necessary to maintain correct incentives. Using the information generated by the other fund's returns, fund owners can indeed reduce the informational rent of the fund manger even further. As a consequence, however, they become dependent on the other fund's incentive system. Mutual, though not formally coordinated, deviation of the two fund managers by neglecting their information-gathering task will strictly increase their payoffs. Moreover, investing with probability one-half either risky or safe in case 1, or only safe in case 2, will not be easily detected by the fund owners, since the observed returns of the funds would be consistent with correct behavior of the managers in some contingencies. It would be a difficult statistical exercise for the fund owners to conclude that both managers have been inactive even after observing the returns of repeated investment decisions. We would therefore argue that in the game between the two fund managers there is a strong presumption that relative performance contracts for both managers will induce shirking.

For the first type of case, if we take seriously the notion that managers act independently and cannot communicate to coordinate explicitly their actions, it seems natural to focus on the (unique) mixed strategy over the strategy combinations (s_3, s_4) and (t_3, t_4) that survives iterative deletion of weakly dominated strategies. Tedious but straightforward calculation using the figures that appear in Table 4 shows that if the managers play this equilibrium in their subgame, their expected utility is greater than $v\ (=10)$ while the principals expected return is negative. This is even easier to see in the example of case 2, where if both managers shirk and unconditionally invest in the safe asset, this yields a certain gross return of 0. The managers are thus paid the contracted amount 66.41 for sure which in turn yields a *guaranteed* loss of -66.41 for each of the two groups of fund owners.

The second case demonstrates that, as a consequence of relative-performance-based remuneration schemes, an outcome is possible where both fund managers choose not to seek information about the investment opportunities and invest in the riskless asset. This may be interpreted as the 'conservative' behavior suggested in the literature concerning managed funds that was discussed in the introduction.

6. The value of relative performance contracts for fund owners

The previous section has shown that relative performance contracts, if introduced without coordination between the fund owners, may well give rise to multiple equilibria in the managers subgame. Furthermore, for the particular parameterization of the model in the last section, 'shirking' and then either investing unconditionally in the safe asset or mixing one's choice between the safe and the risky assets, is always a strategy that survives the iterated deletion of weakly dominated strategies for both managers. The equilibrium strategy that each relative performance contract is intended to implement, however, never survives the procedure of the iterated deletion of weakly dominated strategies.

Since the two groups of fund owners lose if they both set relative performance contracts and the managers play a 'shirking' equilibrium in the ensuing subgame, one may wonder whether the fund owners would not find it optimal simply to ignore the potential information from a fund's relative performance in order to avoid the negative consequences.[10] This leads us to consider the equilibrium of the complete game whose sequence of actions was outlined in Section 2.

Recall that the structure of moves for the two groups of fund owners is very simple. If a fund owner offers an optimal non-relative performance contract, i.e., one that solves the contract problem in Section 3 with the additional constraint that $x_i(\cdot, 0) \equiv x_i(\cdot, a)$ for all possible differences a in the performances of the two funds, then this will guarantee the work effort of the manager. For a fund owner, given the belief that the other fund's manager is following the investment strategy (1, 0), a relative-performance contract (RP), however, clearly dominates a non-relative performance contract (NRP), because the information contained in the observed performance of the rival fund is valuable. Hence, a fund owner using an RP contract, while the other fund owner relies on a NRP contract, will make the larger profit. On the other hand, if both owners use RP contracts, then

[10] Appendix B shows that the NRP contract is indeed the optimal contract for fund owners to set their manager such that it is a (weakly) dominant strategy for that manager to gather information and follow investment strategy (1, 0). Also, it is clear that if one fund has set a NRP contract for its manager then a RP contract will be optimal for the owners of the other fund. Hence, for this section, we will limit attention to the NRP and RP contracts.

Table 6
Fund owners' payoff matrix

		Fund owner 2	
		RP	*NRP*
Fund owner 1	*RP*	U_1 (RP, RP), U_2 (RP, RP)	U_1 (RP, NRP), U_2 (RP, NRP)
	NRP	U_1 (NRP, RP), U_2 (NRP, RP)	U_1 (NRP, NRP), U_2 (NRP, NRP)

both will be worse off than with NRP contracts, because shirking of both managers is the only equilibrium behavior in the managers' subgame that survives iterated deletion of weakly dominated strategies.

Subsuming the managers' equilibrium play, the entire game can be reduced to a one-shot simultaneous move by the two groups of fund owners. The matrix given in Table 6 represents the structure of this game.

These payoffs can be ranked as follows:

$$U_1(RP, NRP) > U_1(NRP, NRP) = U_1(NRP, RP) > U_1(RP, RP),$$

$$U_2(NRP, RP) > U_2(RP, NRP) = U_2(NRP, RP) > U_2(RP, RP).$$

Notice that this game has the structure of a battle-of-the-sexes game with the following two asymmetric pure and unique mixed strategy equilibria:[11]

1. an equilibrium where the owners of the first fund use a relative performance contract and those of the second fund rely on a standard contract, (RP, NRP);
2. an equilibrium where the owners of the second fund use the relative performance contract while those of the first fund rely on a standard contract, (NRP, RP); and
3. both groups of fund owners randomly choose between the relative and non-relative performance contract with fund 1 placing weight

   $$\frac{U_2(NRP, RP) - U_2(NRP, NRP)}{U_2(NRP, RP) - U_2(RP, RP)}$$

   on RP, and fund 2 placing weight

   $$\frac{U_1(RP, NRP) - U_1(NRP, NRP)}{U_1(RP, NRP) - U_1(RP, RP)}$$

   on RP.

[11] Unlike the classic battle-of-the-sexes game, players in this game wish to do the *opposite* of what the other does. Binmore (1992, p. 39) coyly dubs this the Australian battle of the sexes.

Notice that as the strategy *NRP* for a group of fund owners gives the same payoff irrespective of the type of contract offered by the other fund the equilibrium payoff in the mixed strategy equilibrium must give that payoff. That is, the equilibrium payoff for the fund owners in the mixed-strategy equilibrium of the above game in which relative performance contracts are available is the same as for the game in which fund owners were only able to offer their managers non-relative performance contracts.

The structure of the game makes it quite clear that each group of fund owners has an incentive to exploit the benefits of a relative performance contract if the other group does not try to do the same. In particular, it cannot be a stable situation that both funds offer their manager the same type of contract. Thus, one should observe a battle between funds to provide the incentives for their managers at lower cost by using more or less sophisticated contracts. Copying the behavior of the other fund can however never be an equilibrium. There is, of course, the possibility of coordinated behavior of the fund owners by playing a correlated strategy. For example, by switching simultaneously from the strategy combination (*RP*, *NRP*) to the strategy combination (*NRP*, *RP*) on observation of a common random signal the fund owners could achieve a symmetric expected return which exceeds the expected return from the single fund contract. There remain, however, doubts whether this is the appropriate answer to the problem. If fund owners can coordinate their behavior, then it appears hard to argue that they should not be able to design jointly a relative performance contract for both managers as in Holmström (1982) and to resolve the multi-equilibrium problem as suggested in Ma et al. (1988). If instead, we view the fund owners as acting independently, then the mixed strategy equilibrium seems focal with respect to that assumption which ironically entails that the expected outcome for fund-owners, given the availability of relative performance contracts, is exactly the same as they would reap if they could only offer non-relative performance contracts.

Although fund owners are indifferent between the mixed-strategy equilibrium outcome and the payoff from the contract combination (*NRP*, *NRP*), the managers are better off than they would be if only non-relative performance contracts were on offer. In that case their expected utility is simply their reservation utility v, while for the mixed-strategy equilibrium, in the event that both funds offer relative performance contracts they can expect a return greater than v. In the mixed-strategy equilibrium, the 'social gain' that arises through the more efficient use of information that occurs when the contract offer combination is either (*NRP*, *RP*) or (*RP*, *NRP*), more than offsets the loss when no information is gathered in the shirking equilibrium that ensues from the contract offer combination (*RP*, *RP*). But this net social gain is entirely expropriated (in an ex ante sense) by the fund managers.

7. Conclusions

Managed funds have an increasingly important influence on asset trade in financial markets. It has been observed that these funds often follow broadly similar investment strategies, even in periods of crisis such as the stock market crash of 1987, raising fears that fund managers pursue suboptimal investment strategies. In particular, investment strategies appear more correlated than implied by optimal risk-spreading. There are, of course, many possible explanations for this behavior and these explanations often are not mutually exclusive. Performance pressure from competition for investors' funds, for example, has been advanced as an explanation (Leland et al., 1997) for seemingly coordinated behavior between fund managers.

At the same time, the importance of relative performance contracts in reducing the costs of asymmetric information has been theoretically established (Holmström, 1982). While the empirical question of whether relative performance contracts actually provide a low-cost solution to agency problems has not been settled, these contracts have been widely implemented through complex remuneration packages for managers based on the relative performance of their companies.

In this paper we show that strongly correlated behavior and incentive provisions in fund manager contracts can be related problems. Relative-performance contracts provide the correct incentives, at least cost for fund owners given that other fund managers behave correctly, so that with these contracts there is an equilibrium where all fund managers behave as desired by the fund owners. However, least-cost contracts for the owners mean that informational rents are low for the managers. There are alternative equilibria where managers extract more of the informational rents by mutually deviating from the behavior intended by the fund owners. These equilibria typically dominate the equilibrium where managers act as owners want them to behave.

It follows from these observations that there is no symmetric pure-strategy contract equilibrium for fund owners. All funds can neither remunerate their managers successfully by relative-performance contracts, because such contracts cannot be expected to implement the desired behavior; nor rely on independent non-relative performance contracts because, while implementing the desired behavior, such contracts would not do so at the lowest cost possible. Only asymmetric pure-strategy equilibria exist. One fund may reduce its incentive costs by offering its manager a relative performance contract so long as the other fund retains a non-relative performance contract. This implication of relative-performance contracts in the context of fund-management incentives has not been recognized before.

These theoretical observations have important implications for managed-fund investment strategies. Since equilibrium contracts are asymmetric, one fund has higher incentive costs than the other in order to achieve the desired investment policies. This may induce fund owners who have to bear the higher incentive costs to also use relative-performance contracts. If this were to happen, the managerial incentive equilibrium can be expected to break down and inefficient similar investment strategies could be observed.

Would fund owners detect deviation from the intended behavior? Not necessarily, since the strategy the managers choose would be optimal under some contingencies. Of course, fund owners may question their managers' behavior once they observe unchanging investment behavior for many periods. To analyze the fund owners' detection possibilities properly would require a truly dynamic model of the interaction over time between fund owners and their respective fund managers, which is beyond the scope of this paper.

How do these observations relate to the efficient-market hypothesis in its strong form? As Grossman and Stiglitz (1980) have demonstrated, small positive returns on information-gathering activities may be compatible with the strong efficient market hypothesis. Thus, one could argue that informationally inefficient coordinated behaviour of fund managers would create new incentives to gather information. In our model, the incentive for information gathering exists for the fund owners who, after all, have instructed their managers to do exactly this. Since these fund owners may not detect their managers' mutual deviations from the intended behavior, there is no presumption in this model that deviations of the managers will provide extra incentives to gather information.

The last few remarks go beyond the formal argument of the model presented in this paper. In addition, many other important issues related to fund management policies have to remain unanswered in this paper. Relative performance of investment or pension funds is also important for a funds' ability to attract new investors and to maintain their investors' loyalty. In another paper (Eichberger et al., 1997), we focus on the need of fund managers to attract investment as a disciplining device for alleviating moral-hazard problems. In that paper, the amount of funds available for investment becomes the strategically important variable. In contrast, here we have chosen to keep the amount to be invested constant in order to strengthen our argument about the incentives from relative-performance contracts. We view the impossibility of symmetric relative-performance contracts as a major and novel contribution of this paper.

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Appendix A: Proofs

Proof of Lemma 4. The following four lemmata establish this result.

Lemma A.1. In any solution of the relative performance contract problem, the (IR) constraint is binding.

Proof. Suppose x_1^* is a solution with

$$\Pr(H)V_1(1, x_1|H, t_1) + \Pr(L)V_1(0, x_1|L, t_1) - e > v.$$

As $u(\cdot)$ is continuous, there exists $\varepsilon > 0$ such that

$$\begin{aligned}&\Pr(B\cap h\cap H)\cdot u[x_1^*(-1,0)-\varepsilon] + \Pr(B\cap l\cap H)\cdot u[x_1^*(-1,-1)-\varepsilon]\\&\quad + \Pr(G\cap h\cap H)\cdot u[x_1^*(1,0)-\varepsilon] + \Pr(G\cap l\cap H)\cdot u[x_1^*(1,1)-\varepsilon]\\&\quad + \Pr(B\cap h\cap L)\cdot u[x_1^*(0,1)-\varepsilon] + \Pr(l\cap L)\cdot u[x_1^*(0,0)-\varepsilon]\\&\quad + \Pr(G\cap h\cap L)\cdot u[x_1^*(0,-1)-\varepsilon] - e > v.\end{aligned}$$

Hence $x_1'(\cdot)$, where $x_1'(\cdot) := x_1^*(\cdot) - \varepsilon$, satisfies all the constraints but entails a lower expected cost to the fund owners, contradicting the initial hypothesis that $x_1^*(\cdot)$ was a solution. □

Lemma A.2. Either γ_1 and/or γ_2 is strictly positive.

Proof. Assume the contrary, i.e. $\gamma_1 = \gamma_2 = 0$. Hence from Eqs. (3)–(9) and (IR)

$$u[x_1^*(-1,0)] = u[x_1^*(-1,1)] = \cdots = u[x_1^*(0,0)] = v + e.$$

But such a payment scheme contradicts both work constraint inequalities (WC1) and (WC2). □

Lemma A.3. γ_1 is strictly positive.

Proof. Assume the contrary, i.e. $\gamma_1 = 0$ and so by Lemma A.2 $\gamma_2 > 0$. From Eqs. (3)–(6) we have $x_1^*(0,1) = x_1^*(0,-1) = x_1^*(0,0) = x_0^*$. From Eqs. (3)–(9) it follows that $x_1^*(-1,0)$, $x_1^*(-1,-1)$, $x_1^*(1,0)$ and $x_1^*(1,1)$ are all strictly less than x_0^*. But then inequality (IC1) fails to hold. □

Lemma A.4. γ_2 is strictly positive.

Proof. Assume the contrary, i.e. $\gamma_2 = 0$ and so by Lemma A.2, $\gamma_1 > 0$. From Eqs. (7)–(9) we have $x_1^*(-1,0) = x_1^*(-1,-1) = x_1^*(1,0) = x_1^*(1,1) = x_1^*$.

From Eqs. (3)–(9) it follows that $x_1^*(0,1)$, $x_1^*(0,-1)$ and $x_1^*(0,0)$ are all strictly less than x_1^*. But then inequality (IC2) fails to hold. □

This completes the proof of Lemma 4. □

Appendix B

Consider that the owners of fund 1 wish to ensure that their manager always chooses to gather investment information and invest appropriately regardless of the activities of the manager of the second fund. Let t_3 denote the investment strategy (0, 0) where the manager always chooses to invest all the funds in the riskless asset, and t_4 denote the investment strategy (1, 1) where the risky asset is always chosen. The contract design problem for the owners of fund 1 is as given in Section 3, subject to the addition of four extra work constraints:

$$\gamma_3\text{: } \Pr(H)V_1(1, x_1|H, t_3) + \Pr(L)V_1(0, x_1|L, t_3) - e \geq V_1(0, x_1|t_3), \quad \text{(WC3)}$$

$$\gamma_4\text{: } \Pr(H)V_1(1, x_1|H, t_3) + \Pr(L)V_1(0, x_1|L, t_3) - e \geq V_1(1, x_1|t_3), \quad \text{(WC4)}$$

$$\gamma_5\text{: } \Pr(H)V_1(1, x_1|H, t_4) + \Pr(L)V_1(0, x_1|L, t_4) - e \geq V_1(0, x_1|t_4), \quad \text{(WC5)}$$

$$\gamma_6\text{: } \Pr(H)V_1(1, x_1|H, t_4) + \Pr(L)V_1(0, x_1|L, t_4) - e \geq V_1(1, x_1|t_4). \quad \text{(WC6)}$$

For example, (WC3) states that the manager of fund 1 should have an incentive to gather information and invest appropriately, rather than following the strategy (0, 0) even when the manager of fund 2 is following the (0, 0) investment strategy.

Lemma C.1. The work constraints (WC3)–(WC6) imply (WC1) and (WC2).

Proof. From the proof of Lemma 3, the work constraints (WC1)–(WC6) can be written as:

$$\Pr(H)V_1(1, x_1|H, t_1) - e \geq \Pr(H)V_1(0, x_1|H, t_1), \quad \text{(WC1')}$$

$$\Pr(L)V_1(0, x_1|L, t_1) - e \geq \Pr(L)V_1(1, x_1|L, t_1), \quad \text{(WC2')}$$

$$\Pr(H)V_1(1, x_1|H, t_3) - e \geq \Pr(H)V_1(0, x_1|H, t_3), \quad \text{(WC3')}$$

$$\Pr(L)V_1(0, x_1|L, t_3) - e \geq \Pr(L)V_1(1, x_1|L, t_3), \quad \text{(WC4')}$$

$$\Pr(H)V_1(1, x_1|H, t_4) - e \geq \Pr(H)V_1(0, x_1|H, t_4), \quad \text{(WC5')}$$

$$\Pr(L)V_1(0, x_1|L, t_4) - e \geq (L)V_1(1, x_1|L, t_4). \quad \text{(WC6')}$$

Note that (WC1′) and (WC2′) can be rewritten as:

$$\Pr(l|H)V_1(1, x_1|H, t_3) + \Pr(h|H)V_1(1, x_1|H, t_4) - \frac{e}{\Pr(H)}$$

$$\geq \Pr(l|H)V_1(0, x_1|H, t_3) + \Pr(h|H)V_1(0, x_1|H, t_4),$$

$$\Pr(l|L)V_1(0, x_1|L, t_3) + \Pr(h|L)V_1(0, x_1|L, t_4) - \frac{e}{\Pr(L)}$$

$$\geq \Pr(l|L)V_1(1, x_1|L, t_3) + \Pr(h|L)V_1(1, x_1|L, t_4).$$

It immediately follows that (WC1′) is implied by (WC3′) and (WC5′), and that (WC2′) is implied by (WC4′) and (WC6′). □

Solving the fund owners' optimization problem subject to (IR), (WC3)–(WC6) shows that setting an uncontingent contract with $x_1(-1, 0) = x_1(-1, -1)$, $x_1(1, 0) = x_1(1, 1)$ and $x_1(0, 1) = x_1(0, -1) = x_1(0, 0)$ and $\gamma_3 = \gamma_5, \gamma_4 = \gamma_6$ satisfies the first-order conditions. In other words, if the owners of fund 1 wish to make investment strategy s_1 a dominant strategy for their manager, then the optimal contract does not involve relative performance evaluation.

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